TFPR: Dispersion and Cyclicality*

Russell Cooper† and Ozgen Ozturk‡

March 31, 2024

Abstract

This paper studies the determinants of the dispersion and cyclicality of TFPR, a revenue based measure of total factor productivity. Recent business cycle models are built upon the assumption of countercyclical dispersion in TFPQ, a quantity based measure of total factor productivity, based on evidence of countercyclical dispersion in TFPR. But, these are very different measures of productivity. The distribution of TFPR is endogenous, dependent upon exogenous shocks and the endogenous determination of prices. An overlapping generations model with monopolistic competition and state dependent pricing is constructed to study the factors that shape the TFPR distribution. The focus is on three key data patterns: (i) countercyclical dispersion of TFPR, (ii) countercyclical dispersion of price changes and (iii) countercyclical frequency of price adjustment. The analysis uncovers two interesting scenarios in which these moments are matched. One arises in the presence of shocks to the dispersion of TFPQ along with a negatively correlated change in the mean of TFPQ. The second arises if the monetary authority responds to shocks to the dispersion of TFPQ by "leaning against the wind". The findings are robust to the introduction of non-CES household preferences. Due to state contingent pricing, the model is nonlinear. Simple correlations mask these nonlinearities of the underlying economy.

1 Motivation

There is considerable evidence that the dispersion of TFPR, a revenue based measure of total factor productivity, is countercyclical.[1] Despite the central role of this pattern in disciplining models of aggregate fluctuations, the determinants of TFPR dispersion and its cyclicality are not well understood in the literature. The point of this paper is to evaluate alternative hypotheses regarding the countercyclicality of the dispersion in TFPR.

*Comments from David Berger, Edouard Challe, Ian Dew-Becker, John Haltiwanger, Matthias Kehrig, Immo Schott, Stephen Terry, and Jonathan Willis as well as those from referees and the editor are greatly appreciated.

†Department of Economics, European University Institute, russellcoop@gmail.com

‡Department of Economics, European University Institute and Department of Economics, Oxford University, ozgen.ozturk@economics.ox.ac.uk

[1] See the evidence and discussion in, for example, Kehrig (2011), Bachmann and Bayer (2014), and Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). The evidence is presented as changes in the distribution of total factor productivity and/or the correlation in the dispersion of total factor productivity with a measure of economic activity. Bachmann and Bayer (2014) provide complementary evidence from German data.

Our approach builds on a simple decomposition. TFPR is measured as the product of TFPQ, a quantity based measure of firm productivity, and prices. From this, the variance of the log of TFPR is given by the sum of the variance of the log of TFPQ, the variance of the log of prices and the covariance of these two variables.[2] The distribution of TFPR is dependent upon both the exogenous distribution of TFPQ and the endogenous distribution of prices.

Clearly a direct source of dispersion in TFPR is the cyclical dispersion in TFPQ. So it might be that cyclical variations in TFPR dispersion come directly from cyclical variations in the dispersion of TFPQ. This argument is problematic in two ways. First, the well-known Oi-Hartmann-Abel effect suggests that increases in dispersion of productivity increases aggregate productivity and hence output through the reallocation of factors of production.[3] Second, it might seem that the countercyclical dispersion in TFPQ comes from the aggregate effects of uncertainty, as in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). But, contrary to that argument, Berger, Dew-Becker, and Giglio (2020) and Dew-Becker and Giglio (2023) find no persuasive evidence that variations in uncertainty are a driver of aggregate fluctuations. This casts doubt on the argument that countercyclical *ex ante* uncertainty over increased dispersion of TFPQ coupled with the realization of higher dispersion lies behind the countercyclical dispersion in TFPR.

Accordingly, our approach looks at the behavior of price and their correlation with productivity as additional sources of variation in the dispersion of TFPR.[4] The endogeneity of prices allows a variety of other shocks, both monetary and real, to impact the dispersion of TFPR. Additional moments, focusing on price setting, are included in the analysis to allow us to discriminate between these sources of exogenous fluctuations.

One main finding is that counter-cyclical movements in the dispersion of TFPQ are necessary to generate counter-cyclical dispersion in TRPR. But, they are not sufficient. In order to match data patterns, the shock to the dispersion of TFPQ must be coupled with either a drop in the mean of TFPQ or a monetary contraction. Without these additional shocks, the dispersion of TFPR would be procyclical, reflecting the power of the Oi-Hartmann-Abel effect.

To highlight the role of pricing in a tractable way, an overlapping generations model with monopolistic competition and state dependent pricing is constructed to study the factors that shape the TFPR distribution.[5] The state dependent pricing structure allows us to consider the contributions of both the extensive margin (to adjusts prices or not) as well as the intensive margin to determining both the dispersion of prices and their covariance with productivity. The response of the extensive margin to shocks is itself highly nonlinear, with adjustment more likely for large (in absolute value) shocks. The response of the pricing decisions, on both the extensive and intensive margins, are key to cyclical variations in the covariance between firm

[2] This follows directly from the definition of (logorithm) TFPR: tfpr=ln(p)+ tfpq. This and alternative measures of TFPR are discussed in Foster, Haltiwanger, and Syverson (2008).

[3] This comes from Oi (1961), Hartman (1972) and Abel (1983).

[4] There is a parallel here with Berger and Vavra (2019). That paper distinguishes between changes in distributions and responsiveness. In our formulation the distribution of TFPQ is the exogenous variation and responsiveness is reflected in price determination in response to a wide range of variations, including the dispersion in TFPQ.

[5] The gains from adopting the overlapping generations model are explained in detail below.

prices and their productivity.

The quantitative focus is on three key data patterns: (i) countercyclical dispersion of TFPR, (ii) countercyclical dispersion of price changes and (iii) countercyclical frequency of price adjustment.[6] The analysis uncovers two scenarios in which these moments are matched, highlighting nonlinearities in responses to shocks.

Due to price setting behavior, TFPR dispersion responds to large number of aggregate shocks, including variations in the money supply, the distribution of idiosyncratic demand, the mean of productivity and the distribution of idiosyncratic productivity. Analyzed independently, these shocks generate cyclical movements in the dispersion of TFPR, both through effects on the dispersion of prices and, perhaps more interestingly, through the covariance of prices and the firm specific productivity shock. Our first main finding is that the model moments produced from these sources of fluctuations, individually, do not match data patterns. In particular, variations in the dispersion of TFPQ shocks alone do not produce countercyclical dispersion in TFPR.

We then consider combinations of these sources of variations. Our second set of findings points to two settings, both involving fluctuations in the dispersion of TFPQ that match data patterns. First, a shock to the dispersion of TFPQ combined with a perfectly negatively correlated shock to the mean of TFPQ, creates the comovements documented from the data. Essentially the dispersion of TFPR is driven by the dispersion of TFPQ while output movements depend more on the mean of TFPQ. The combination of shocks provides a mechanism that drives a wedge between the dispersion of TFPR and that of TFPQ. Combining these two shocks is crucial: an increase in the dispersion of TFPQ alone cannot capture the empirical pattern of countercyclical dispersion in TFPR. This result is supportive of findings in the literature. In order to avoid negative correlation between consumption and investment in the face of an uncertainty shock, Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), combine a shock to the dispersion of TFPQ with a reduction in average TFPQ. Vavra (2014) employs a variation of this specification. But, again, the mechanisms are different. We do not rely on uncertainty shocks. Further, our shocks relate to the distribution of TFPQ not the endogenous distribution of TFPR so that the pricing decisions impact the TFPR distribution.

There is a second case in which the monetary authority responds to exogenous variations in the dispersion of TFPQ, thus creating a comovement with the money shocks. If the monetary authority "leans against the wind", i.e. tightens monetary policy when output is high, in the face of shocks to the dispersion of TFPQ, then the data patterns of countercyclical dispersion in TFPR, the dispersion of price changes and the frequency of price adjustment emerge. As with the combination of the negatively correlated shocks to the mean and dispersion of TFPQ, the tighter monetary policy offsets the output increase induced by more dispersed productivity.

Section 6 looks at extensions and additional properties of the model economy. First, explore an alternative calibration driven more by macro considerations, close to Vavra (2014). Our main findings remain. Second,

[6]The latter two are the focus of Vavra (2014) as well. But that analysis does not take into account the distinction between TFPQ and TFPR.

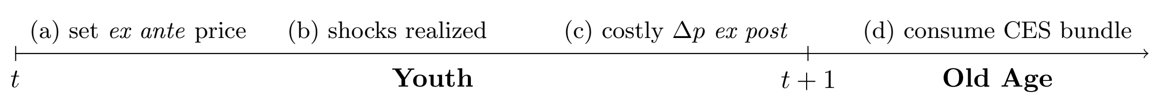
we enrich household preferences to allow state dependent elasticity of demand using the Kimball aggregator.[7] We study how this alternative representation of preferences impacts the pricing decisions of sellers as well as the determinants of the countercyclical dispersion in TFPR. Our main findings remain though the magnitudes of responses of prices, particularly the covariance of prices and idiosyncratic productivity, are lower. Third, we highlight the nonlinear properties of the equilibrium response to shocks, which comes from the U-shaped price adjustment hazard. For this, we compute conditional correlations to indicate how variables comove as a function of the aggregate state (expansion or contraction). Finally, we introduce uncertainty into the framework, distinguished from *ex post* changes in dispersion. We find no effects of uncertainty.[8]

2 State Dependent Pricing Model

We study the determination of the distribution of TFPR in an infinite horizon overlapping generations model with differentiated products and market power. Agents live for two periods, youth and old age.[9] Generation t young agents produce and, when old, these agents consume a basket of goods produced by the next generation of young producers. Saving occurs through the holding of fiat money. The quantity of fiat money is stochastic, representing monetary shocks.[10]

As noted earlier, this is admittedly not the standard framework for the analysis of state dependent pricing. The benefit of the model is the simplicity of the choice problems coupled with an equilibrium analysis that allows experiments without the introduction of unexpected shocks to money and/or the distributions of productivity and demand. This point is central since understanding how the economy responds to shocks is the key question of the paper. Answering this in a fully articulated stochastic economy is therefore necessary.

Figure 1: Time Line: Generation t



The sequence of choices is shown in Figure 1. Generation t young agents set a price *ex ante*, prior to the determination of any shocks but dependent on the history of the economy, summarized in equilibrium by

[7] We appreciate the suggestion of a referee to add this specification of the model. The baseline model assumes CES utility.

[8] This is in line with the relative importance of dispersion relative to "wait and see" effects reported in Bachmann, Born, Elstner, and Grimme (2019).

[9] This can be viewed as a model in which agents work in one month and consume in the next. Consumption in both months is ignored for tractability. The key is not the horizon *per se* but the static pricing problem they solve, as discussed below.

[10] This presentation focuses on a version of the model in which there are aggregate shocks to the model supply as well as idiosyncratic shocks to seller productivity and to the cost of price adjustment. The quantitative analysis adds other sources of variation. The extension of the model to include these additional shocks appears in Subsection 2.5 and Appendices A.2 - A.3.

the stock of money inherited from the previous period. This indicated by (a) on the timeline. At point (b) shocks to the aggregate money supply as well as to idiosyncratic productivity and idiosyncratic menu costs are realized. Given these realizations, sellers have an option of *ex post* price adjustment, indicated by point (c). This is the step that generates heterogeneous price setting, both on the extensive margin (to adjust the *ex ante* price or not) and in the event of adjustment, the intensive margin choice of what price to set.

There are a couple of features of the model economy worth highlighting. First, the price setting stage is interdependent in that the optimal price of one seller depends on the *ex post* price of the adjusters as well as the *ex ante* price of the non-adjusters.

Second, the *ex post* decision on price adjustment depends on the realization of all shocks. In this way, the dispersion of the distribution of productivity shocks impacts the frequency of adjustment and thus the real effects of money shocks.

Third, the inclusion of two forms of idiosyncratic shocks, one to productivity and the other to the adjustment costs, creates an interesting tension in the adjustment decision. A seller with a very large productivity shock might be induced to adjust the *ex ante* price but may draw a high adjustment cost and thus not reset its price. This tension has implications for the equilibrium effects of money shocks as the selection into price adjustment depends on all of these shocks. Further, for our purposes, the relationship between the exogenous TFPQ distribution and the endogenous TFPR distribution depends on the price setting behavior of sellers.

Fourth, as in Lucas (1972), in the absence of price stickiness, there would be a stationary rational expectations equilibrium in which money was neutral. This is because money transfers are made to the old in proportion to money holding earned in youth. And, as in that paper, the analysis rests on the coexistence of real and nominal shocks. But, in our setting the friction of costly price adjustment replaces his assumption of imperfect information.[11]

2.1 Choice of Old Agents

Lifetime utility is represented by $u(c) - g(n) = \frac{c^{1-\sigma}}{1-\sigma} - g(n)$. Here c is a CES aggregator given by $c = \left(\sum_i c^{i\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$, with $\varepsilon > 1$.[12] The function $g(\cdot)$ is increasing and convex in hours worked, with $0 \leq n \leq 1$. As we shall see, both the substitutability between products as well as the curvature in the disutility of work play important roles in the pricing decisions of young agents, particularly the choice of *ex post* adjustment.

When old, agents take their money holdings from income earned in youth and allocate it across goods to maximize $u\left(\left[\sum_i (c^i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right)$, subject to a budget constraint of $\sum_i c^i p^i = M$ where M is their nominal income and p^i is the money price of good i.[13]

[11] Of course, in his model the real shock was to the fraction of sellers in a particular market while we focus on productivity shocks.

[12] We normalize the number of young agents and thus products to 1. With the CES assumption, markups are constant. For now, this puts aside another potentially interesting interaction between the level of economic activity and prices. Sub-section 6.2 studies a richer sense of preferences based upon the Kimball aggregator.

[13] To simplify the notation, the time subscript is repressed. The money holdings come from income earned in youth as money is the store of value in this economy. Many other general equilibrium models, such as Dotsey, King, and Wolman (1999), impose

For these preferences, the demand for good i is given by

$$c^i = d(p^i, P, M) = \left(\frac{p^i}{P}\right)^{-\varepsilon} \frac{M}{P}. \tag{1}$$

Here P is an aggregate price index defined as $P = \left(\sum_i (p^i)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$. Note that the only shock to demand is from variations in the stock of money, M.

Let $V(\frac{M}{P})$ be the value of the solution to the optimization problem of an old agent with nominal income of M with prices given by P. Given the definition of c,

$$V(\frac{M}{P}) = u\left(\left[\sum_i \left((\frac{p^i}{P})^{-\varepsilon}\frac{M}{P}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right) = u\left(\left[\sum_i \left((\frac{p^i}{P})^{-\varepsilon}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \frac{M}{P}\right) \tag{2}$$

with P given above. From this, the marginal value of nominal income is given by $V_M = \frac{u'(c)}{P}$.

At this point, these are generic demands and values for an old age given nominal income and prices. These values summarize the outcome of the choice problem for old agents in period t at point (d) of the time line in Figure 1. We will take this structure and use it to study the choices of young agents in the OG framework, summarizing the utility they obtain when old through $V(\frac{M}{P})$.

2.2 Choice of Young Agents

We start with the pricing decisions of generation t young agents. When young agents choose the price of their product *ex ante*, they take into account the option, at a fixed cost, of adjusting their price *ex post*. These are points (a) and (b) in Figure 1. Since this is a model of a menu rather than a quadratic cost at the micro-level, the *ex ante* price will influence the frequency of adjustment but not the *ex post* price conditional on adjustment.

As is common in the sticky price literature, see for example Galí (2015), sellers are assumed to meet the demand forthcoming at their price. Thus the prices they set will determine their nominal income in youth, given the aggregate state.

This nominal income is held over time in the form of money to purchase consumption goods when old. Holdings of money are altered through monetary policy. Thus in our framework, money holdings and monetary policy interventions are made explicit.

To study the pricing choice, consider the *ex post* decision of generation t sellers.[14] If they choose to adjust, these sellers choose a price $\tilde{p}$ to solve

$$W^a(z_t, M_{t-1}, x_t, P_t) = max_{\tilde{p}} E_{x_{t+1},P_{t+1}} V(R(\tilde{p}, P_t, M_t)x_{t+1}/P_{t+1}) - g(\frac{d(\tilde{p}, P_t, M_t)}{z_t}). \tag{3}$$

Here the demand, denoted $d(\tilde{p}, P_t, M_t)$ and specified in (1), is the spending of the old agents on the

money demand. In Golosov and Lucas (2007), money is in the utility function.

[14]That is, we solve the agents problem at point (b) and use this solution to study the *ex ante* problem at point (a).

product of this seller. The function $V(R(\tilde{p}, P_t, M_t)x_{t+1}/P_{t+1})$ is given by (2) with, in that notation, $M = R(\tilde{p}, P_t, M_t)x_{t+1}$ being the nominal revenue earned as a seller in period t supplemented by the period $t+1$ money shock and $P = P_{t+1}$, the period $t+1$ aggregate price level.

Since this decision is made *ex post*, the value and the price depend on the current state: (z_t, M_{t-1}, x_t, P_t). Here z_t is the current idiosyncratic productivity shock, M_{t-1} is the aggregate money supply inherited from the previous period, x_t is the money shock and P_t is the aggregate price level, determined in equilibrium as described below.

There is also a seller specific menu cost, denoted F, that influences whether adjustment occurs or not but not the price selected given adjustment. The adjustment cost is written as a utility loss. This specification has a convenient property that the optimal price is independent of the adjustment cost. So, the extensive margin of adjustment will depend on the realized menu cost and idiosyncratic productivity but the intensive margin does not so that the price dispersion of adjusters reflects only heterogeneity in z_t.

In this formulation, the menu cost F has a cumulative distribution function denoted $\Omega(\cdot)$. The inclusion of stochastic menu costs weakens the selection effect, i.e. the dependence of the pricing decision on z. As we shall see with the calibrated model, this implies that the probability of price adjustment is an increasing function of the absolute value of the idiosyncratic technology shock but it is not a step-function. That is, there are no bounds on z such that price adjustment occurs iff z is outside those bounds.

Notice that the price set by these sellers is independent of any price they may have set *ex ante* so that the *ex ante* choice does not appear in the state space. Importantly, once the cost of adjustment is incurred, the price reflects both the monetary shock and seller specific productivity. In this sense, there is an underlying complementarity at work. If a seller pays an adjustment cost to respond to one type of shock, then the marginal cost of responding to another type of shock is zero. This is important for the analysis that follows as it explains why price dispersion and thus TFPR dispersion is influenced by monetary policy.

With the production function of $y = zn$, the labor input of the seller is given by $\frac{d(\tilde{p}, P_t, M_t)}{z_t}$. As the seller meets all demand, the labor input varies inversely with productivity.

The first-order condition is

$$E_{x_{t+1}, P_{t+1}} \left(u'(c_{t+1}) x_{t+1} \frac{d(p_t, P_t, M_t)(1-\varepsilon)}{P_{t+1}} \right) = g'(\frac{d(p_t, P_t, M_t)}{z_t}) \left(-\varepsilon \frac{d(p_t, P_t, M_t)}{p_t z_t} \right). \tag{4}$$

Denote this *ex post* optimal price by $p_t = \tilde{p}(z_t, M_{t-1}, x_t, P_t)$ for a seller with realized productivity z_t.

This is the standard condition for optimal price setting, equating marginal revenue with marginal cost.[15] But in this overlapping generations model, marginal revenue is determined by the expected marginal utility of the future consumption that can be acquired with the additional money income. And that income is itself

[15] To understand this condition in a static setting, let $d = (\frac{p}{P})^{-\varepsilon} y$ be the level of produce demand if the seller sets the price p and the aggregate price is P and the level of real spending is y. So $d_p = -\varepsilon \frac{d}{p}$. Further, revenue is given by $R = pd = p^{1-\varepsilon}(\frac{1}{P})^{-\varepsilon} y$. Hence $R_p = (1-\varepsilon)d$. The left side of (4) is the product of R_p and $\frac{u'(c_{t+1})x_{t+1}}{P_{t+1}}$. The right side is the product of d_p and the marginal disutility of work, $g'(\frac{d(p_t, P_t, M_t)}{z_t})\frac{1}{z_t}$.

impacted by future monetary policy, through the stochastic transfer x_{t+1}.

Alternatively, if the seller does not adjust, then expected lifetime utility is given by:

$$W^n(z_t, M_{t-1}, x_t, P_t, \bar{p}) = E_{x_{t+1}, P_{t+1}} V(R(\bar{p}, P_t, M_t) x_{t+1}/P_{t+1}) - g(\frac{d(\bar{p}, P_t, M_t)}{z_t}). \tag{5}$$

Here, expected utility depends on the preset price, $\bar{p}$.

Given this, consider the *ex ante* choice. When this price is set, the young agent just knows the money supply from the past. Let $W^{xa}(M_{t-1})$ be the value to a young agent of setting the price *ex ante*. The value is given by:

$$\begin{aligned} W^{xa}(M_{t-1}) \quad &= \quad max_{\bar{p}} E_{(z_t, x_t, x_{t+1} P_t, P_{t+1})}[(1 - \Omega(F^*(\Omega_t)))W^n(z_t, M_{t-1}, x_t, P_t, \bar{p}) + \\ &\int_0^{F^*(\Omega_t)} W^a(M_{t-1}, x_t, P_t) - F] d\Omega(F) \end{aligned} \tag{6}$$

where $F^*(z_t, M_{t-1}, x_t, P_t)$ is the critical menu cost in state (z_t, M_{t-1}, x_t, P_t) such that price adjustment occurs iff $F \leq F^*(z_t, M_{t-1}, x_t, P_t)$. Let $\bar{p}(M_{t-1})$ denote the optimal *ex ante* choice.

2.3 SREE

The analysis is based on a stationary rational expectations equilibrium (SREE) with valued fiat money.[16] The current aggregate state is represented as (M, x) where M is the inherited money supply and x is the current shock, so that the current money supply is Mx. At the individual supplier level, productivity and the cost of price adjustment are the two elements in the idiosyncratic state: (z, F). At this point of the analysis, the distribution of the idiosyncratic shocks is fixed and thus not in the state vector. An equilibrium is defined and characterized given that distribution.

There are four state dependent functions to be determined. The *ex ante* price set knowing only M is denoted $\bar{p}(M)$. The *ex post* price set by sellers who choose to adjust their price is given by $\tilde{p}(M, z, x)$, indicating the price depends on both the realized money shock and productivity. There is a critical level of the adjustment cost, $F^*(M, x, z)$, such that adjustment occurs iff $F \leq F^*(M, x, z)$. Finally, the *ex post* money price of goods, $P(M, x)$, clears the goods market.

Definition 1 A SREE is a set of functions $(\bar{p}(M), \tilde{p}(M, z, x), F^*(M, x, z), P(M, x), W^n(M, x, z), W^a(M, x, z))$ such that:

- $\bar{p}(M)$ solves the *ex ante* pricing problem given the state dependent price index $P(M, x)$;

$$\bar{p}(M) = argmax_p E_{x,z,x'} V((R(p, P(M, x), Mx)x')/P(Mx, x')) - g(\frac{d(p, P(M, x), Mx)}{z}). \tag{7}$$

[16] The more general SREE -including shocks to the distribution of idiosyncratic productivity, as well as other aggregate shocks- is presented in Appendix A.2. To avoid confusion with terminology, stationarity means that these functions of the state are not indexed by time.

for all M.

- $\tilde{p}(M,x,z)$ solves the *ex post* pricing problem:

$$\tilde{p}(M,x,z) = argmax_p E_{x'} V\left((R(p,P(M,x),Mx))x'/P(Mx,x')\right) - g(\frac{d(p,P(M,x),Mx)}{z}) \tag{8}$$

given the state dependent price vector, $P(M,x)$, for all (M,x,z).

- At the critical adjustment cost, $F^*(M,x,z)$, the seller is just indifferent between adjusting and not:

$$F^*(M,x,z) \equiv W^n(M,x,z) - W^a(M,x,z) \tag{9}$$

for all (M,x,z), with $W^a(M,x,z)$ given by:

$$W^a(M,x,z) = E_{x'} V\left((R(\tilde{p}(M,x,z),P(M,x),Mx))x'/P(Mx,x')\right) - g(\frac{d(\tilde{p}(M,x,z),P(M,x),Mx)}{z}) \tag{10}$$

and $W^n(M,x,z)$ given by

$$W^n(M,x,z) = E_{x'} V((R(\bar{p}(M),P(M,x),Mx))x'/P(Mx,x')) - g(\frac{d(\bar{p}(M),P(M,x),Mx)}{z}). \tag{11}$$

- $P(M,x)$ is the aggregate price index in state (M,x) given by:

$$P(M,x) = [E_z(1-\Omega(F^*(M,x,z)))\bar{p}(M)^{1-\varepsilon} + E_z(\Omega(F^*(M,x,z))\tilde{p}(M,x,z)^{1-\varepsilon})]^{\frac{1}{1-\varepsilon}} \tag{12}$$

where $d(\bar{p}(M),P(M,x),Mx) = \left(\frac{\bar{p}(M)}{P(M,x)}\right)^{-\varepsilon} Y$ and $d(\tilde{p}(M,x,z),P(M,x),Mx) = \left(\frac{\tilde{p}(M,x,z)}{P(M,x)}\right)^{-\varepsilon} Y$. Here $Y = \frac{Mx}{P(M,x)}$ is the equilibrium determined real value of money holdings.

2.4 Equilibrium Properties

This section briefly describes properties of a SREE, both at the aggregate and individual seller level. These properties are made more explicit in the quantitative analysis.

2.4.1 Money Non-Neutrality

There are two main properties of a SREE that are verified in the analysis that follows.

Proposition 1 *There exists a SREE in which: (i) real quantities are independent of M since all prices set ex ante and ex post are proportional to M and (ii) real quantities are not independent of x.*

First, the inherited money supply is neutral: i.e. prices are proportional to M and all real quantities are independent of M. Formally, this amounts to guessing and verifying that there is a SREE in which both

$\bar{p}(M)$ and $\tilde{p}(M, x, z)$ are proportional to M. From this all relative prices and thus quantities demanded (and thus supplied) are independent of M.

The second property is money non-neutrality. If prices were not costly to adjust, i.e. the distribution of F was degenerate at $F = 0$, then there would exist a SREE with prices proportional to Mx. In this case, real quantities would be independent of the current money supply, Mx. But, in the presence of non-degenerate menu costs, as long as some sellers choose not to adjustment their prices *ex post*, a SREE with prices proportional to Mx cannot exist simply because the preset price, $\bar{p}$, must be independent of x.[17]

2.4.2 Productivity Measures

Returning to the theme of productivity measures, the difference between TFPQ and TFPR is straightforward to characterize. Here, z corresponds to the TFPQ measure of productivity. It is exogenous to the seller. The variable $\frac{zp}{P}$ is TFPR, where $p \in \{\tilde{p}(M, x, z), \bar{p}\}$ reflects the seller's pricing choice and P is the aggregate price index.[18] Though the distribution of TFPQ is exogenous, the distribution of TFPR is endogenous as prices are set by sellers. Thus the distribution of TFPR responds to shocks insofar as sellers adjust prices in response to those shocks.

The price stickiness as well as the limited reallocation of labor across production sites help to shape the distribution of TFPR. To illustrate, consider a static, flexible price version of the model where the production function is $q = zn^{\alpha}$ and ε parameterizes the elasticity of inverse demand, $p(q) = q^{-\varepsilon}$. From the first order condition with respect to n, if then marginal cost of labor is ω:

$$(1 - \varepsilon)\alpha n^{(-\alpha\varepsilon + \alpha - 1)} z^{1-\varepsilon} = \omega.$$

At $\alpha = 1$, this condition becomes $(1 - \varepsilon)n^{-\varepsilon} z^{(1-\varepsilon)} = \omega$ which holds for all z. By definition, $TFPR = pz = q^{-\varepsilon} z = z^{1-\varepsilon} n^{-\varepsilon}$, where the last equality relies on $\alpha = 1$. Using the first order condition, TFPR for any seller is therefore given by $\frac{\omega}{1-\varepsilon}$, independent of the realization of z. So, in this limiting case of constant returns to scale and flexible prices, variations in the distribution of TFPQ would not impact the distribution of TFPR as the latter is degenerate.[19]

In our model, both price stickiness and increasing marginal cost along with labor immobility will contribute to the non-degenerate distribution of TFPR.[20] Specifically, the state dependent pricing shapes the distribution of TFPR. For sellers not adjusting their prices, $TFPR = \bar{p}z$ and there is no covariance between

[17]Formally, this requires that the support of menu costs be large enough so that even if all other sellers adjust their prices *ex post*, the remaining seller, for any x, will have a high enough adjustment cost so that adjustment will not occur. See Ball and Romer (1991) for a discussion of this related to multiplicity of equilibria.

[18]Since TFPQ is measured directly in simulated data, there is no need to infer TFPR from revenue and thus no discussion of output or revenue factor shares. See the discussion of these measurement issues in Decker, Haltiwanger, Jarmin, and Miranda (2019).

[19]See Hsieh and Klenow (2009) on this point.

[20]Further, with more general preferences, the markup will be not constant as the elasticity of demand will vary with the price. This is discussed in sub-section 6.2. Asker, Collard-Wexler, and De Loecker (2014) also discuss the non-degenerate distribution of TFPR and focus on capital adjustment frictions. Further, Asker, Collard-Wexler, and De Loecker (2014) study the distributions of TFPR and the MPK across countries.

the *ex ante* price and realized marginal cost. For sellers that choose to adjust, as in the simple static example, $TFPR = \frac{\varepsilon}{\varepsilon-1} g'(\frac{d(\cdot)}{z})z$. This expression comes from the *ex post* first order condition so that the price is a markup over marginal cost. In our economy, marginal cost reflects the marginal disutility of work. As before, if costs were linear, then TFPR would be independent of productivity conditional on adjustment.

From this, there are two elements that together determine the *ex post* distribution of TFPR. First, unlike the Calvo model there are state dependent variations in the fraction and selection of adjusters. This movement on the extensive margin impacts the distribution of TFPR. And since the adjustment hazard is highly non-linear, this creates non-linearity in the dispersion of TFPR. Second, on the intensive margin, the distribution of z will impact the distribution of TFPR through the sellers who choose to adjust their price because of increasing marginal cost.

2.4.3 Seller Choices

In equilibrium, aggregate real output is given by: $Y(x) = \frac{Mx}{P(M,x)} = \frac{x}{\varphi(x)}$, where, using the first part of Proposition 1, $P(M,x) = M\varphi(x)$. Thus the response of output to money shocks will depend on $\varphi(x)$, in the absence of other aggregate shocks. This function summarizes the responses by sellers to monetary shocks. It captures both the extensive margin of adjustment, i.e. the fraction of sellers resetting their price *ex post*, as well as the intensive margin of the optimal price to set.

The money shock impacts both margins. In terms of adjustment frequency, more extreme shocks generate a higher fraction of sellers choosing to adjust. Further, for those sellers adjusting, the *ex post* will depend on the money shock. But, importantly, it will not be proportional to x. Thus the non-neutrality arises on both the extensive and intensive margins.

There is an important feature of our model that ties directly with the line of research which studies the frequency of price adjustment as a function of a gap between actual and desired prices. Caballero and Engel (2007) discuss this approach and cite numerous related papers. Our model, with its one time price adjustment, fits exactly into that framework.[21] This can be seem from (9), where the difference in the values between adjusting and no adjusting are used to determine the critical adjustment cost. These difference in values is directly related to the gap between the *ex ante* price, $\bar{p}(M)$, and the state contingent *ex post* price, $\tilde{p}(M, x, z)$.[22]

2.5 Additional Shocks

Thus far the analysis includes only a single aggregate shock. This was simply to enhance the transparency of the presentation. Introducing additional sources of aggregate and idiosyncratic shocks into this framework

[21] The cost of static pricing is that there is no endogenous persistence of a shock to the money supply. From the estimated model Smets and Wouters (2007), it seems that the degree of indexation of prices, which is a proxy for the persistence in prices, does not play a main role in their results. Rather, the presence of habit formation and capital adjustment costs are key elements of the dynamic response. In other models, nominal variables are automatically reset at the start of a period, thus eliminating persistence through the pricing channel. See, for example, the discussion in Section IV. D of Vavra (2014).

[22] This is explored in the quantitative analysis of the linear quadratic economy.

is important for understanding the determinants of the dispersion in TFPR.

Appendix A.2 presents the more general economy in which there is an aggregate state, S, that includes shocks to the money supply, variations in the distribution of z and relative demand shocks.[23] The optimization problems of agents as well as the definition of equilibrium is directly extended to this enhanced environment. It is the basis of the quantitative analysis that follows.

Two shocks to the distribution of TFPQ are studied. One is the traditional TFPQ shock in which the mean of the z distribution, denoted μ_Q, is stochastic. In this case, the output of a seller becomes $y = \mu_Q zn$. The second, which follows the motivation of the paper is a shock to the dispersion of z, denoted $disp_Q$, holding the mean fixed.

Finally, the model is extended to incorporate idiosyncratic demand shocks.[24] This provides a direct shock to the dispersion of TFPR, through demand, and independent of the dispersion in TFPQ. These are modelled as seller specific shifts in demand. As discussed below, these shocks differ from the idiosyncratic productivity shocks, particularly when prices are sticky. In terms of aggregate shocks, we study mean preserving spreads in the distribution of demand shocks, denoted $disp_D$. Variations in the mean level of nominal spending are studied through the money shocks.

Through all of these extensions of the stochastic framework, the basic structure of the model and the insistence on a SREE is maintained. Further, the numerical solution operates directly on the conditions for a SREE, without the need for linear approximations. Given the reliance of the literature on various approximations to a SREE, conducting these experiments in a general equilibrium model provides an alternative approach consistent with the discipline of equilibrium analysis.

3 Quantitative Analysis

The estimation of underlying parameters is best left for an empirical exercise that studies price setting by infinitely lived firms matching high frequency observations on price and quantities. At this point, such an ideal data set is not available.[25] Our goal is more modest and should be considered as an extended quantitative example allowing us to focus on the determination of the distribution of TFPR in an equilibrium model.

That said, the quantitative version of the OG pricing model has features of the standard macroeconomic pricing models, including both the Calvo model and state dependent pricing problems. In the Calvo model, as in the OG structure, the probability of price adjustment and the price set conditional on adjustment, are independent of the previously set price. Further, in some specifications, such as Christiano, Eichenbaum,

[23]The inclusion of the relative demand shocks is motivated by the findings of the importance of this source of variation in Hottman, Redding, and Weinstein (2016). See Sedlacek and Ignaszak (2021) for a discussion of demand vs technology shocks as drivers of firm growth and innovation.

[24]The augmented model is discussed in Appendix A.1. See Eslava and Haltiwanger (2020) for discussion of a similar specification.

[25]An exception is Garcia-Marin and Voigtländer (2019) who study producitivity of exporters and are able to isolate TFPQ from prices. But apparently there is not enough time series variation to study aggregate fluctuations.

and Evans (2005), price setters who do not adjust get to freely reset prices based upon past inflation. This added feature further reduces the role of history for price setting. In the OG model, this is captured by period t price setters choosing a price that is proportional to the inherited money supply.

Further, as discussed in Klenow and Malin (2010), existing evidence suggests that for individual sellers the likelihood of price adjustment at a particular point in time is independent of the time since last adjustment. Though allowing full state dependence (conditional on paying an adjustment cost), our model also has this history dependent feature as the choices of sellers in period t does not depend on prices in the past.

Price setting in this model also reproduces familiar patterns of state dependent price adjustment. That is the model generates pricing rules for sellers that retain the essential features of the more standard infinitely lived agent specifications. This is made clear in the discussion of the pricing behavior of sellers below.

The calibration of the model serves two purposes. First, it sets the basis for the quantitative assessment of the cyclical properties of the distribution of TFPR. Second, as the model includes both demand and technology shocks, the analysis contributes to the ongoing discussion of the relative importance of these sources of variation.

3.1 Calibration

The quantitative analysis rests upon a linear-quadratic economy: $u(c) = c, g(n) = \frac{n^\phi}{\phi}$, where ϕ is the elasticity of labor supply.[26] For the baseline, $\phi = 2$. Varying this elasticity impacts the shapes of marginal cost and thus the benefits of price adjustment.

The key parameters govern the price adjustment costs and the dispersion of idiosyncratic productivity. These are calibrated so that the steady state of our economy, i.e. the SREE without aggregate shocks, matches a set of moments.[27]

Even in the absence of aggregate shocks, the model produces a rich set of cross sectional moments given the presence of idiosyncratic shocks to productivity, idiosyncratic demand shocks and menu costs. The model calibration related to the distributions of the shocks rests on evidence related to the distributions of TFPQ and TFPR as well as the frequency of price adjustment.

The parameters characterizing the distribution of menu costs come directly from Dotsey and Wolman (2019) and are shown in the top panel of Table 1.[28] Note that this parameterization allows a free price adjustment with probability slightly over 5%. A period is a month.

The linear-quadratic specification leaves three parameters, $(\varepsilon, \sigma_z, \sigma_d)$ to be determined. To do so, we use moments from Vavra (2014) and Foster, Haltiwanger, and Syverson (2008) as shown in Table 2.

The frequency of price adjustment are taken from Vavra (2014), where the model is calibrated on a

[26] Appendix A.3 characterizes the SREE for the linear quadratic preferences.

[27] Aruoba, Oue, Saffie, and Willis (2023) use related micro facts for their calibration of a model that studies markups and pass-through. An alternative calibration is studied in sub-section 6.1.

[28] These are discussed in detail in Appendix A.4.3. In principle, the parameters of the menu cost distribution could have been estimated as well. In practice, this proved difficult along two dimensions: (i) matching moments and (ii) finding an equilibrium. Thus we focus more on the stochastic processes and the elasticity of substitution.

Table 1: Parameterization

Parameter	Value	Description
Menu Cost Distribution		
ψ	0.053	Probability of zero menu cost
$\bar{F}$	0.033	Upper bound on menu cost
ω	41.9	Curvature parameter
ν	2.8	Curvature parameter
Utility Parameters		
ϵ	2.37	Elasticity of substitution between products
ϕ	2	Elasticity of labor supply
Idiosyncratic Productivity Shock		
σ_z	0.0378	Standard Deviation
Idiosyncratic Taste (Demand) Shock		
σ_d	0.0069	Standard Deviation

monthly frequency. For Vavra (2014), the standard deviation of TFPR on a monthly frequency is set to match the annual measure from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). Vavra (2014) reports the standard deviation of the innovation, the persistence of the shock and the probability of a change in his Table III of calibrated parameters. In our model, all young sellers draw a shock from an ergodic distribution. Thus we infer the standard deviation of TFPR from the standard deviation of the innovation and the persistence reported by Vavra (2014).

Table 2: Matching Moments

Moment	Data	Model	Source
$disp_R$	0.102	0.103	Vavra (2014)
$disp_Q/disp_R$	1.181	1.181	Foster, Haltiwanger, and Syverson (2008)
$freq_{\Delta p}$	0.110	0.127	Vavra (2014)

Note: This table shows basic moments computed from time series averages and the steady state of our model using the parameters in Table 1. All variables are logarithms except for frequency of price adjustment.

Given our focus on the distinction between TFPQ and TFPR, independent observations on these objects is quite informative. From Foster, Haltiwanger, and Syverson (2008) annual estimates, we take 1.181 as the ratio of the dispersion in TFPQ to the dispersion in TFPR. From experiments, it seems that this ratio is not influenced by time aggregation: simulating a higher frequency model and time aggregating preserves this ratio.[29]

As seen in Table 2, the calibration matches the moments well, though we do not quite reproduce the

[29] This was studied through simulation in a partial equilibrium setting with Calvo price setting.

frequency of price adjustment reported in Vavra (2014).[30] The calibrated value of ε is below the level of other studies, such as Vavra (2014) and Golosov and Lucas (2007). Also, the dispersion of demand shocks is significantly lower than the dispersion of technology shocks, in contrast to Hottman, Redding, and Weinstein (2016) and Eslava and Haltiwanger (2020).

3.2 Seller Choices

This section illustrates the quantitative properties of the seller's choices for the linear-quadratic economy. Among other things, it makes clear that the policy functions from the overlapping generations model have properties quite similar to those produced by an infinitely lived seller. Throughout we focus on the response to idiosyncratic shocks, leaving aggregate shocks to the next section.

3.2.1 Pricing

As in the traditional state dependent pricing model, prices are adjusted only for sufficiently large shocks and the region of adjustment depends on the adjustment costs. In addition, because of the presence of stochastic menu costs, the probability of adjustment, given z, lies strictly in $(0, 1)$ unless z is in one of the tails. These properties is illustrated in Figure 2 in the steady state of our model.

Two perspectives are shown in the figure. In the top two panels, the adjustment probability depends on the idiosyncratic productivity, on the left, and idiosyncratic demand, on the right. The adjustment probability is U-shaped indicating that adjustment is more likely for extreme values of these shocks.[31]

The bottom panel provides an alternative but equivalent expression of the adjustment probability. Here the horizontal axis measures the difference between the log of the price the seller would set if adjustment was free and the log of the *ex ante* price. This measure, often called the price gap, is the foundation for the extensive research, from Caballero and Engel (1993) and Caballero and Engel (2007), on the relationship between adjustment rates and (price) gaps.[32] The likelihood of price adjustment as a function of the price gap inherits the U-shaped patterns of the responses of adjustment to technology and demand shocks.

This is a natural metric for this analysis. In the overlapping generations model, the price gap is not an approximation for the actual state but is a summary statistic for the gains to adjustment, to be weighed against the costs. That is, the structure of this model fits exactly with the requirements of the approach that summarizes the state through a price gap.[33]

As we shall see, the representation of adjustment rates as a function of the price gap is more convenient. Once aggregate shocks are introduced, the mapping from the idiosyncratic shocks to the likelihood of price adjustment will become state dependent. But, as made clear in Caballero and Engel (1993) and used as

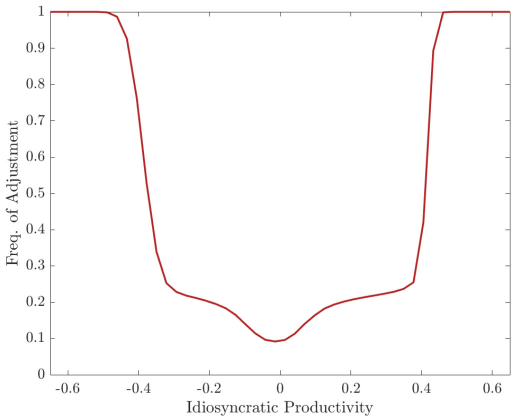
[30] One point of difference is that Vavra (2014) excludes temporary adjustments.

[31] The adjustment rate does not go to 1 in panel b because of the limited domain of the demand shock displayed.

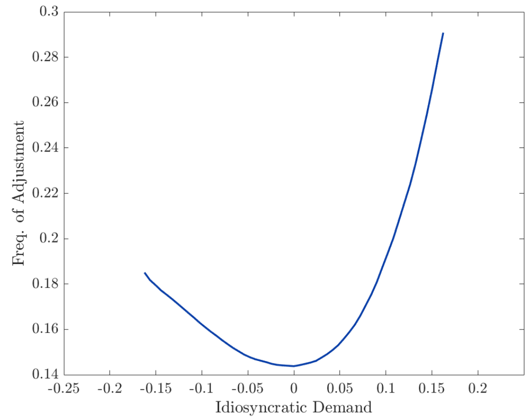
[32] This is used in Vavra (2014) too.

[33] This is because of the limited time horizon. In an infinite horizon setting, the target price is often defined as the optimal price in the absence of adjustment costs assuming integrated shocks. Here no assumptions on the distribution of future variables are needed and, of course, permanent versus temporary opportunities to adjust are equivalent.

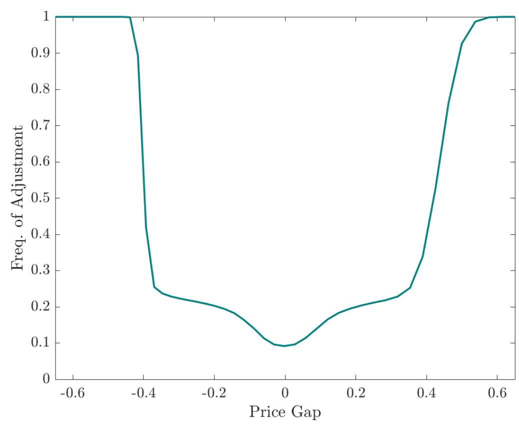
Figure 2: Adjustment Probabilities



(a) Productivity



(b) Demand



(c) Price Gap

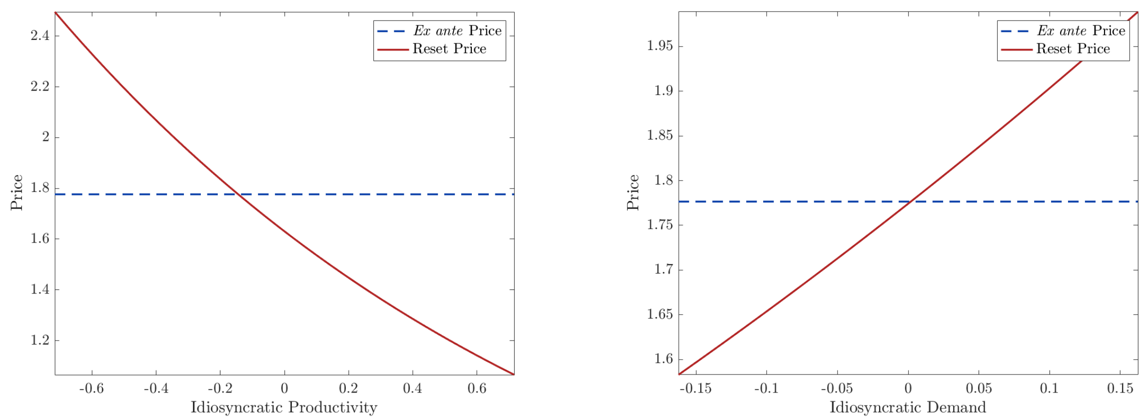
Note: These figures show the adjustment rates of a seller in a steady state on idiosyncratic productivity, demand and the price gap, defined as the difference between the log of the optimal *ex post* price of the seller and the price set *ex ante*: $ln(p^*(\cdot)) - ln(\bar{p})$.

well in Vavra (2014), variations in idiosyncratic as well as aggregate states are neatly summarized by the price gap so that the adjustment probability is not a state dependent function of the price gap. Instead, the aggregate shocks impact the distribution of the price gaps across sellers. Interacting with the non-linear hazard, the distribution of these gaps will have aggregate implications.

The fact that the model economy produces this shape for the adjustment rate is important for two reasons. First, it confirms that state dependent pricing in the overlapping generations model produces patterns that are similar to other models. There is nothing special about the OG pricing structure with respect to the shape of this adjustment hazard.

Second, as the analysis develops, the aggregate economy will display non-monotonic responses to various types of shocks. Those patterns can be traced back to the U-shaped adjustment rate. Because the equilibrium of the model is characterized directly, that is without log-linear approximations, the aggregate non-linearities produced through this hazard will be sustained.

Figure 3: Reset Price



(a) Dependence on Productivity **(b)** Dependence on Demand

Note: These figures show the dependence of the reset price on idiosyncratic productivity and idiosyncratic demand.

Conditional on adjustment, the optimal price of the seller satisfies the first-order condition, (8), in the steady state where $x = 1$ with probability 1. In the calibrated model, the *ex post* optimal price is a decreasing function of productivity and an increasing function of demand. This is shown in Figure 3.

3.2.2 Output and Employment Responses

This subsection studies the employment and output response to idiosyncratic demand and productivity shocks.[34] The results are enriched by the endogenous pricing decision of sellers.

Table 3 reports regression results estimated from simulated data for experiments characterized by the type of shocks: (i) idiosyncratic productivity shocks and, (ii) idiosyncratic demand shocks. The dependent variable is either the (log of) producer employment or output. The columns indicate the response of sellers who did and did not choose to adjust their price.

For the employment column, the negative coefficient for the non-adjusters arises from the fact sellers who do not adjustment their price decrease employment since demand is given. For the adjusters, the effect of productivity on employment is always positive. Because the adjusters raise their price in the face of a demand shock, their employment (and output) response is less than the non-adjusters.

For adjusters, output expands with either productivity or demand shocks. For non-adjusters, idiosyncratic productivity shocks have no output effects again since demand is given. Non-adjusters, given the price,

[34] Aggregate output in the model is measured as the ratio of the stock of money to the price index. From experiments, this is quite close to deflating by a CPI using the steady state equilibrium to construct the basket of goods. For the demand shocks, $z = 1$.

expand output to meet demand.

Table 3: Dependence of Employment and Output on Productivity and Demand

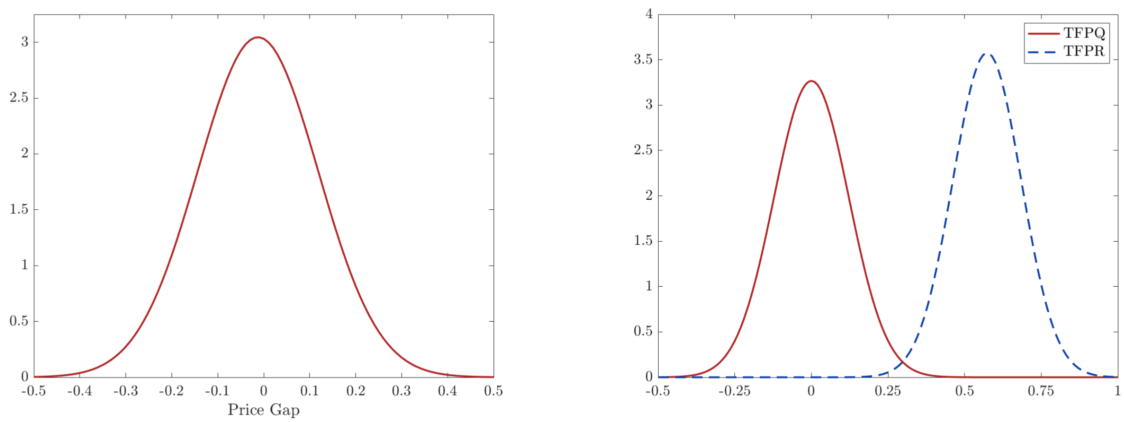
	Employment		Output	
	Adj.	*No Adj.*	*Adj.*	*No Adj.*
Productivity	0.235	-0.573	0.825	0
Demand	0.406	1.367	0.406	1.367

Note: This table shows the effects of idiosyncratic productivity (z) and demand (d) on producer-level employment and output conditioning on price adjustment status.

3.3 Distributions

In the absence of aggregate shocks, the interesting features are the distributions of prices, the price gaps and TFPR, given the distribution of TFPQ. The pricing itself has an extensive margin, to adjust or not, as well as an extensive margin regarding the response of the reset price to the idiosyncratic state z.

Figure 4: Aggregate Implications



(a) Gap Distribution: Steady State **(b)** Productivity Distributions: Steady State

Note: This figure shows the price gap and productivity distributions in the steady state.

Figure 4a presents the steady state distribution of the price gap, allowing both idiosyncratic productivity and demand shocks. It is centered around zero and reflects the underlying distribution of the idiosyncratic productivity shocks. Clearly there are many sellers with relatively small gaps and who, from the adjustment hazard, are unlikely to adjust their price. Those in the tails have a larger gain to adjustment and thus are more likely to adjust. Compared to Figure 2, the distribution puts relatively little weight on gaps which are large enough to warrant adjustment with probability 1. Still there is considerable weight on the nonlinear adjustment rates for intermediate size gaps.

Figure 4b shows the distributions of the two measures of productivity in the steady state. The distribution of TFPQ is given while that of TFPR comes from the interaction of the TFPQ distribution and the pricing choices of sellers. Since prices, contingent on resetting, are decreasing in productivity, there is less dispersion in TFPR than in TFPQ, as seen in Figure 4b. The distribution of TFPR is shifted to the right of the TFPQ distribution through endogenous prices.

4 Cyclicality of TFPR Dispersion

The model of state dependent prices provides a basis to study the cyclicality of TFPR dispersion. The question is whether the model of price setting can reproduce the countercyclical dispersion in TFPR seen in the data, as well as other pricing facts. This depends both on price setting behavior and exogenous variations. Here the exogenous variations include changes in the dispersion of idiosyncratic productivity, ($disp_Q$), changes in the dispersion of the idiosyncratic demand ($disp_D$), aggregate money shocks (x) and changes in the mean of TFPQ (μ_Q).[35]

It is almost immediate that variations in the dispersion of TFPQ will cause variations in the dispersion of TFPR, incorporating optimal price setting. But what about the other shocks? They operate directly on the $disp_R$ given $disp_Q$. Is there any *ex ante* reason to believe they might lead to countercyclical dispersion in TFPR?

For this, the decomposition of the variance in the log of TFPR (tfpr) is:

$$Var(\text{tfpr}) = Var(\text{tfpq}) + Var(ln(p)) + 2 \times Cov(ln(p), \text{tfpq}). \tag{13}$$

Table 4 shows this decomposition in the data. The "FHS" row shows this decomposition for the data from Foster, Haltiwanger, and Syverson (2008). As noted earlier, the variance of tfpr is less than that of tfpq. This is a consequence of the negative covariance between prices and tfpq. So variations in the dispersion in tfpr can be created either by variations in the log of prices or through their covariance with tfpq. The latter effect is directly related to the emphasis on pricing in this paper.

The second row labeled recessions is based upon but not taken directly from the data since the evidence in Foster, Haltiwanger, and Syverson (2008) does not have a cyclical component. It is constructed as a thought experiment where the increase in the variance of tfpr during a recession is taken from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). By assumption, the variance in tfpq is held fixed. The increased variance in p comes from Table 1 of Vavra (2014). The residual is the covariance of prices and tfpq, which is a key to generating cyclical variations in the dispersion of tfpr.

Leaving aside shocks to the dispersion in tfpq, the challenge is then to find exogenous variations that would create countercyclical dispersion tfpr through an increased dispersion of prices along with an increase in the (absolute) value of the covariance. From this exercise, it seems that shock(s) that can create both increased dispersion in prices as well as a higher (in absolute value) covariance of prices and idiosyncratic productivity

[35] The calibration of these processes is discussed in the Appendix sub-section A.4.2.

Table 4: Variance Decomposition: Data

	$Var(tfpr)$	$Var(tfpq)$	$Var(ln(\text{p}))$	$Cov(ln(\text{p}), tfpq)$
			Data	
FHS	0.0484	0.0676	0.0324	-0.0258
recessions	0.0618	0.0676	0.0506	-0.0282

Note: This table shows the decomposition of the variance of tfpr. FHS data are annual (Foster, Haltiwanger, and Syverson (2008)). The percent changes for recession $var(tfpr)$ comes from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), the recession $var(ln(\text{p}))$ is from Vavra (2014), the $cov(\cdot)$ is solved. Recessions are calculated assuming $disp_Q$ is fixed. All variables are logarithms.

can indeed increase the dispersion in measured tfpr. From our model, variations in these moments come both from the extensive margin of price adjustment as well as the dependence on productivity, conditional on adjustment.

4.1 Main Findings: a Preview

Table 5 summarizes our main findings and serves to organize the more detailed discussion that follows. It displays, by source of variation, the cyclical patterns of dispersion in TFPR, the dispersion of price changes, and the frequency of price adjustment. For this part of the analysis, a recession (expansion) refers to output below (above) its steady state value.

The table is discussed in detail in this section, first by looking at each shock independently. We then consider some shocks in tandem, as in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) and Vavra (2014). Finally, we allow the monetary authority to respond to variations in the mean and dispersion of TFPQ and study the implications for the dispersion of TFPR.

The results are best evaluated relative to moments from the data. From various studies, $disp_R$ is countercyclical, the dispersion of price changes and frequency of price changes are countercyclical. The negative correlation of output (growth) and $disp_R$ comes from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), Table V.[36] The data entries for the dispersion and frequency of price changes come from Vavra (2014), Table 1. These are monthly regressions with dummies for recessions. The same regressions are run on our simulated data to produce the cyclical variations reported in Table 5. By choice, we do not use correlations to summarize business cycle properties. The model, as suggested by the U-shaped hazard has very non-linear responses to shocks. Looking at these through the lens of correlations leads to the omission of the rich interactions produced by the model.[37]

As seen in this table, there are two settings in which data patterns are qualitatively matched:

[36]Due to the absence of monthly data to measure output and TFPR, this is a quarterly measure from the data. Our simulation results are at a monthly frequency. Kehrig (2011) finds that the correlation of (detrended) output and the dispersion of productivity is -0.293 for non-durables and -0.502 for durables, in Table 2. His Table 4 makes clear that the countercyclicality is robust to various output measures.

[37]Sub-section 6.3 returns to this point and shows correlations conditional on expansions or contractions.

Table 5: Cyclical Variations

Shock	Moments $disp_R$		$disp_{\Delta p}$		$freq_{\Delta p}$	
	Contraction	Expansion	Contraction	Expansion	Contraction	Expansion
Data						
	0.219	0.051	0.090	0.073	0.161	0.149
Individual Shocks						
$disp_Q$	0.114	0.229	0.047	0.184	0.115	0.327
x	0.155	0.172	0.119	0.115	0.286	0.237
$disp_D$	0.180	0.181	0.080	0.080	0.167	0.166
μ_Q	0.179	0.168	0.092	0.098	0.186	0.214
Correlated Shocks						
$disp_Q, \mu_Q$	0.258	0.148	0.239	0.089	0.372	0.182
Leaning Against the Wind						
$disp_Q$	0.164	0.127	0.114	0.084	0.219	0.158
μ_Q	0.155	0.155	0.129	0.128	0.300	0.345

Note: This table shows the cyclical patterns of the dispersion in TFPR, $disp_R$, the dispersion in price changes, $disp_{\Delta p}$ and the frequency of price adjustment, $freq_{\Delta p}$. The moments are displayed as columns, for contractions and expansions. The rows refer to the model economies distinguished by the source of exogenous variation as developed in the sections below.

(i) when there is a negative correlation between dispersion in TFPQ and the mean of TFPQ and (ii) when the monetary authority leans against the wind in the face of dispersion shocks. For the other cases, including fluctuations driven by shocks to the dispersion of TFPQ alone, data patterns are not matched.

4.2 Dispersion in TFPQ Shocks

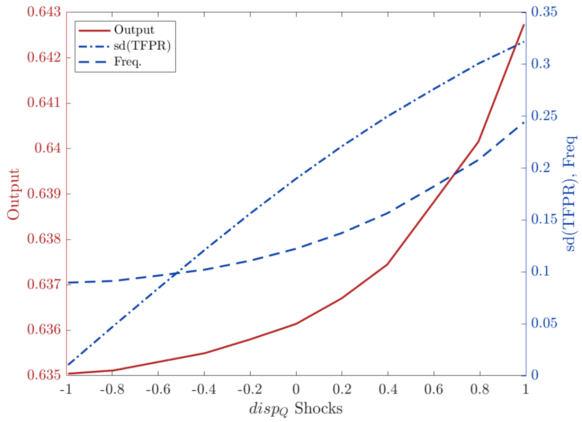
The analysis of countercyclical variation in TFPR dispersion starts with an obvious hypothesis: variations in $disp_Q$ drive the cyclicality of $disp_R$. To order for this explanation to be consistent with data patterns, it must be that: (i) increased dispersion in TFPQ creates increased dispersion in TFPR and (ii) increased dispersion in TFPQ causes economic downturns. We demonstrate that the model does not produce these patterns: variations in the dispersion of TFPQ do not generate countercyclical fluctuations in the dispersion of TFPR.

Specifically, here we study the effects on $disp_R$ of an increase in $disp_Q$, modelled as a mean preserving spread in the distribution of z.[38] To be clear, the effects highlighted here come from realized changes in the distribution of TFPQ, there is no uncertainty effect in the analysis.[39]

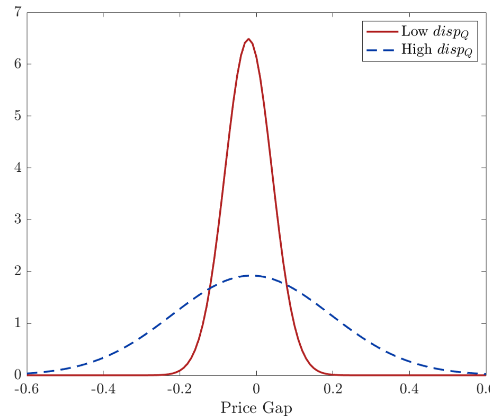
[38]As discussed in Appendix sub-section A.4.2, these variations are about the same size as those explored in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018).

[39]By construction, the mean of the distribution is held fixed.

Figure 5: $disp_Q$ Shock



(a) Responses



(b) Gap

Note: This figure shows the relationship between output, $disp_R$ and the frequency of price adjustment as well as the price gaps as a function of shocks to $disp_Q$

Variations in $disp_Q$ will impact $disp_R$ in two ways. First, of course, there is the direct effect: given prices, an increase in $disp_Q$ will translate into an increase in TFPR dispersion. Second, pricing behavior will adjust, potentially magnifying (reducing) the effects of the increase in $disp_Q$. The sign and size of this latter effect will depend on the properties of the revenue function and, as emphasized by our model, the pattern of price adjustment.

Figure 5a shows the response of output, the frequency of price adjustment and $disp_R$ in response to variations in $disp_Q$.[40] Clearly output is an increasing function of this dispersion, allowing sellers with high productivity to expand.[41] This is the Oi-Hartmann-Abel effect noted earlier. The frequency of price adjustment itself increases as the increased dispersion in z puts more weight on the tails of the price gap distribution, inducing more price adjustment. This is clearly evident in Figure 5b where the price gap distribution is shown for two levels of $disp_Q$.

Overall, for this case, drawing on Figure 5a and Table 5, $disp_R$ is monotonically increasing in $disp_Q$ and hence in output. A key element is that $disp_Q$ is procyclical in the model. This is not necessarily inconsistent with evidence since the negative correlation found in numerous studies between output and dispersion relates to measured $disp_R$ **not** $disp_Q$. The findings about the cyclicality of the dispersion in price changes and frequency are consistent with Vavra (2014) if $disp_R$ was countercyclical. But the model is inconsistent with the data in terms of the motivating observation of countercyclical dispersion in TFPR. Consequently, from Table 5, the variations in price change dispersion and frequency are counter to the data.

A final comment on this experiment relates to the range of variation in $disp_Q$. Bloom, Floetotto,

[40] Note that in the figures, x-axis represents percentage changes relative to the benchmark $disp_Q$ level, which is calibrated as σ_z from Table 1. Specifically, a value of +0.4 on the x-axis indicates a 40% increase from this mean dispQ level, while -0.4 denotes a 40% decrease.

[41] Importantly, these reallocation effects are hampered by both price rigidity and the immobility of labor.

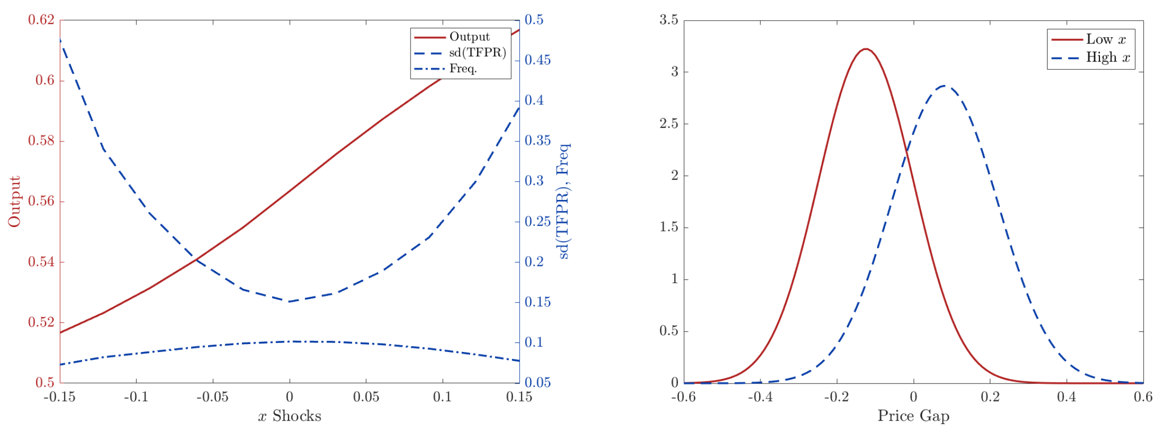
Jaimovich, Eksten, and Terry (2018) parameterize the dispersion shock to TFPQ using the observed variation in the dispersion of TFPR of 4. From Figure 5a, this is roughly a variation in $disp_Q$ +/- 0.4. Focusing on variations within that range, the variations in both the standard deviation in TFPR and the frequency of price changes remain procyclical.

4.3 Money Shocks

A second aggregate shock comes from monetary innovations, x. Due to price rigidities, monetary shocks impact real output. Further, the distribution of TFPR is impacted by monetary shocks, given the distribution of TFPQ, due to both the intensive and extensive margins of price adjustment.

From Table 5, for this source of variation, $disp_R$ is procyclical in contrast to the data. As for the moments characterizing pricing, both the dispersion of price changes and the frequency of adjustment are countercyclical, in line with data patterns.

Figure 6: Money Shock



(a) Responses **(b)** Gap

Note: This figure shows the relationship between output, $disp_R$, the frequency of price adjustment and the price gap as a function of money shocks.

Figure 6a shows the response of output, $disp_R$ and the frequency of price adjustment to monetary shocks: an increase in money growth of 10% leads output to expand by almost 10%.[42] Reflecting price rigidities, output is a monotone function of the innovation to the money supply. As highlighted in Figure 2, the frequency of adjustment is a U-shaped function of the price gap, reflected here in the response to money shocks. Importantly for our analysis, this translates into an inverse U-shaped relationship between $disp_R$ and the money shock. As the shock deviates from its mean value, the frequency of adjustment increases indicating that this nonlinearity is present for relatively small shocks. Of course, price adjustment is much higher in the tails. Since price setters are responding to the common realization of x, there is a reduction in

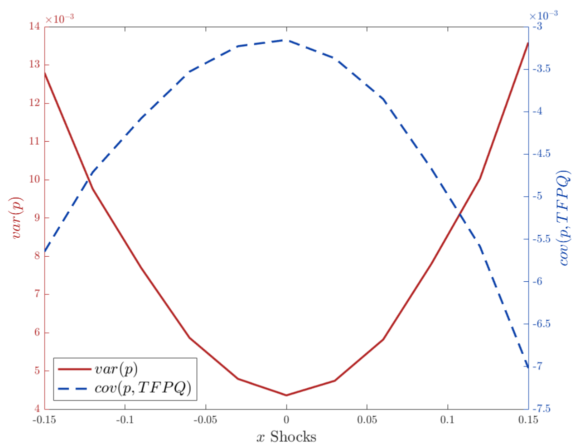
[42]The distribution of money shocks is shown in Appendix Figure A.3. It is bell-shaped around $x = 1$.

the dispersion of TFPR. Though the realized idiosyncratic productivity, z, is independent of x, the selection into adjustment, again using Figure 2, will be those in the tails of the productivity distribution.

The effects of the money shock on the gap distribution is shown in Figure 6b. In contrast to the increased dispersion of the gap distribution from a $disp_Q$ shock, the monetary shock causes a rightward shift. The additional weight on the right tail from a high value of x will increase the frequency of upward price adjustments.

It is useful to understand how a money shock influences the distribution of TFPR. From the decomposition of (log) TFPR in (13), with the dispersion in TFPQ fixed, variations in $disp_R$ come from two sources: (i) changes in the dispersion of prices and (ii) changes in the covariance between prices and productivity. Both of these components are effected by the endogenous price adjustment: they are absent in the Calvo adjustment model. Figure 7 shows how these two cross-sectional moments vary nonlinearily with the money shock. For extreme values of the money shock, the dispersion of prices is higher and the covariance of price and productivity is also higher in absolute value. This reflects the increased frequency of adjustment, as in Figure 6a, as well as the dependence of prices on z for those sellers who choose to reset. This is in keeping with the role of dispersion and covariance brought out in Table 4.

Figure 7: Price Dispersion and Covariance of Prices and Productivity



Note: This figure shows the variance of prices and the covariance of prices and productivity at the micro-level for alternative money shocks.

Overall, as real output increases with the money shock, the model implies that the standard deviation of TFPR is not a monotone function of economic activity when fluctuations are induced by money shocks. It can be lower in recessions and also lower in expansions when the money shocks take relatively extreme values. Thus, the model can produce countercyclical dispersion in TFPR, for a given distribution of TFPQ, conditional on large money shocks. Importantly, the change in the dispersion of prices and their covariance seen in this experiment follows the qualitative pattern of the data, as shown in Table 4.

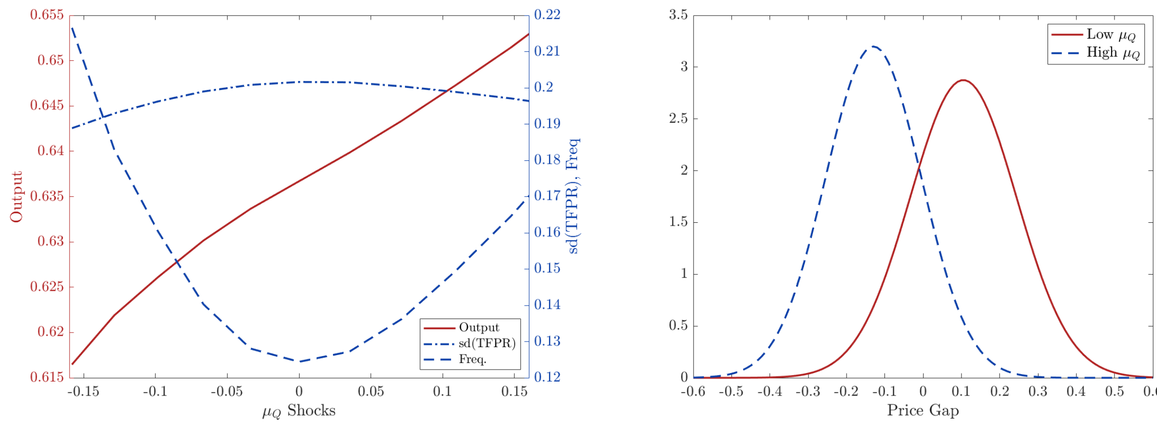
Of course, the empirical importance of the nonlinearities depends on the variability of the shock.[43] The

[43] We appreciate the comments of a referee on this point.

fundamental nonlinearity of the model comes from the frequency of price adjustment and it is operative even if monetary shocks are concentrated close to the mean. But, the response to the dispersion of TFPR in the neighbourhood of the mean is linear and slightly increasing. Further, the output response, for this specification is essentially linear as well.

4.4 Shocks to the Mean of TFPQ

Figure 8: μ_Q Shock



(a) Responses **(b)** Gap

Note: This figure shows the relationship between output, $disp_R$, the frequency of price adjustment and the price gap as a function of shocks to the mean of TFPQ.

The another leading source of variation is the more standard shock to the average productivity, i.e. the mean of TFPQ, denoted μ_Q. As before, the interest is in the cyclicality of the dispersion in TFPR induced by this shock. For now, we study its impact in isolation. Experiments below couple this with a shock to $disp_Q$ as well as a monetary response.

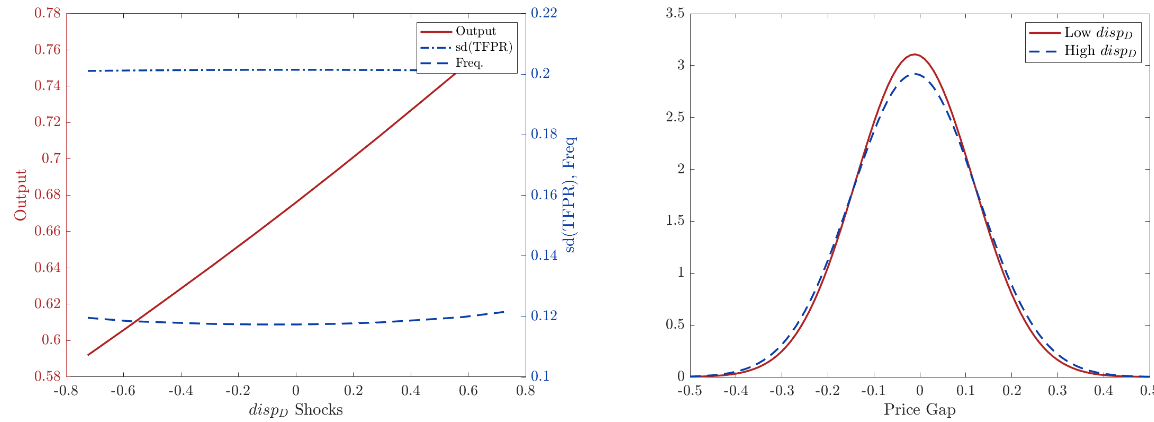
Figure 8a summarizes the findings. As in standard RBC models, output is an increasing function of mean productivity. The frequency of price adjustment is again U-shaped, reflecting the larger gains to adjust for more extreme realizations of mu_Q along with the shift in the price gap distribution, shown in Figure 8b. The dispersion in TFPR is almost flat, decreasing slightly for realizations in the tails where there is more price adjustment.

Thus this case does not produce the data pattern of countercyclical dispersion in TFPR. Further, from Table 5, for this source of variation, both the dispersion of price changes and the frequency of adjustment are procyclical, in contrast with data patterns.

4.5 Dispersion of Demand Shocks

A final source of aggregate variations arises from changes in the dispersion of idiosyncratic demand shocks.

Figure 9: $disp_D$ Shock



(a) Responses **(b)** Gap

Note: This figure shows the relationship between output, $disp_R$ and the frequency of price adjustment as a function of shocks to $disp_D$.

As with variations in $disp_Q$, this is a mean preserving spread in demand shocks. The money shocks can be interpreted as variations in the mean of demand.

As shown in Figure 9a, output increases with demand dispersion, as it did with increased dispersion in TFPQ. In response to increased dispersion in demand shocks, $disp_R$ is slightly countercyclical. This is quite different than the response of $disp_R$ to an increase in $disp_Q$. Part of the explanation lies in the response of output and employment to a demand shock at the producer level, summarized in Table 3. From Table 5, the frequency of price adjustment and its dispersion increase with this shock, so that both of these moments are procyclical.

4.6 Shocks to the Dispersion and Mean of TFPQ

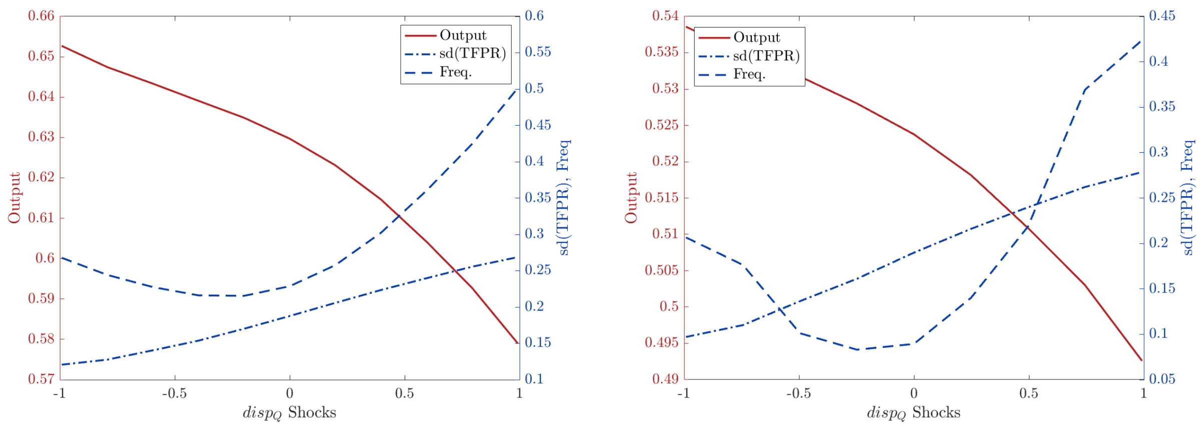
In many studies, such as Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) and Vavra (2014) the shock to dispersion and to the mean of TFPQ are studied jointly. Given the prominence of this case in the literature, it is important to study this case in detail.[44] Here we follow the baseline model in Vavra (2014) and assume the shocks are perfectly negatively correlated: $corr(disp_Q, \mu_Q) = -1$. As the shocks are log normally distributed, the skewness of the cross-sectional distributions of TFPQ is changed by this experiment. Specifically, as μ_Q decreases and $disp_Q$ increases, the skewness increases as well. From Berger, Dew-Becker, and Giglio (2020) this negative covariance of the mean and dispersion of TFPQ is consistent with business cycle patterns.

From Table 5, this is the experiment that brings the model and data patterns closest together. All three moments, $disp_R$, the dispersion of price changes and the frequency of adjustment are

[44]Other combinations were studied without success in matching moments.

countercyclical. Further, as reported in Vavra (2014), the frequency of adjustment and the dispersion in price changes move together, rising in downturns relative to expansions. Intuitively, the variation in μ_Q drives output while the variation in $disp_Q$ dominates in terms of the dispersion of price changes and the frequency of adjustment.

Figure 10: Correlated $disp_Q, \mu_Q$ Shocks



(a) Baseline **(b)** Low Probability of Free Adj.

Note: This figure shows the relationship between output, $disp_R$ and the frequency of price adjustment as a function of shocks to the $disp_Q$. By construction, as the dispersion increases, the mean of TFPQ falls.

The results from Figure 10a illustrates the effect of combining these shocks. The horizontal axis shows $disp_Q$. By construction, as it increases μ_Q decreases. From the graph, the dispersion in TFPR rises with $disp_Q$ while output falls.

This is quite different from the case in which only $disp_Q$ varies, shown in Figure 5a, where output was increasing in $disp_Q$. Further, in contrast to that case, with the combined shock, there is much more response in the frequency of price adjustment to variations in $disp_Q$.

This result does not emerge because of negative comovement between $disp_R$ and $disp_Q$. The dispersion in TFPR increases, albeit modestly, driven by the increase in $disp_Q$. Instead, the decrease in mu_Q has a stronger effect on output than the increase in $disp_Q$. This creates the countercyclical variation in both $disp_Q$ and $disp_R$.

Though the data patterns are matched in this experiment, the magnitudes are not. In particular, the frequency of price adjustment as well as the dispersion of price changes are larger than in the data moments. These can be reduced by setting a lower frequency of free adjustment, the parameter ψ. Reducing that value from 0.053 to 0 lowers the frequency of adjustment below the steady state value but matches the regression dummies for both the frequency of adjustment and dispersion of price changes. This is seen in Figure 10b. Over the range of the $disp_Q$ shocks between $[-0.5, 0.5]$, the frequency of price adjustment lies in $[0.1, 0.2]$, close to the variation found in the data. Over that range, the standard deviation of TFPR increases by about a factor of 2, about half of what is seen in the data based upon the estimates of the cyclical variation in TFPR dispersion from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018).

5 Monetary Feedback Rules

One important theme of the analysis is the nonlinearity in response to monetary shocks produced by the U-shaped frequency of price adjustment. Building on this, we enrich the setting to allow interactions between the shocks, focusing on monetary policy responses. Allowing the monetary authority to link the distribution of x to the aggregate state can alter the cyclicality of $disp_R$. In this way, the implications of the model can be brought closer to some features of the data.

Specifically, suppose that the evolution of the money supply is given by:

$$M_{t+1} = M_t x_{t+1} = M_t[\Phi(s_{t+1})\tilde{x}_{t+1}]. \tag{14}$$

In this specification, the money stock follows the same stochastic process as above, with x_{t+1} representing the period $t+1$ money shock that is not predictable given period t information.[45] But here, the growth of the money supply, $\Phi(s_{t+1})\tilde{x}_{t+1}$, has two components. The first is the feedback rule where $\Phi(s_{t+1})$ allows money growth to depend on the period $t+1$ state of the economy. The second is the money shock, denoted $\tilde{x}_{t+1}$ above.

We focus on two specific cases, distinguished by the source of fluctuations in the aggregate economy. These cases produced variations in the dispersion of TFPR that are qualitatively similar to data moments.[46]

In the first, the monetary authority responds to changes in the dispersion of TFPQ. Let $\overline{disp_Q}$ be the average value of $disp_Q$ and consider

$$\Phi(disp_Q) = \zeta \times (disp_Q - \overline{disp_Q}). \tag{15}$$

In a similar fashion, let $\overline{\mu_Q}$ be the average value of the mean of TFPQ and consider

$$\Phi(\mu_Q) = \zeta \times (\mu_Q - \overline{\mu_Q}). \tag{16}$$

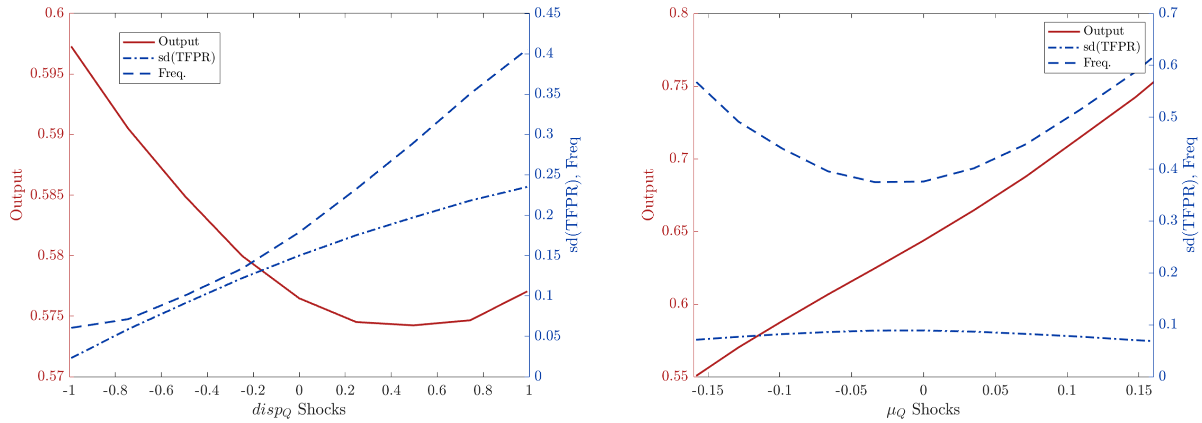
In both formulations, the feedback is characterized by a single parameter, ζ. In contrast to the specification in Tenreyro and Thwaites (2016), the feedback rule is linear so that the nonlinearities are created by the model rather than the conduct of monetary policy.

Given a monetary feedback rule, it is straightforward to extend the analysis of a SREE from Appendix A.3 to include (14). Note that the monetary feedback rule impacts agents both as young price setters and as old agents, both in terms of the distribution of the stochastic transfer and the equilibrium prices they

[45] This is again one of many possible specifications of a feedback rule intended to illustrate this potential channel. An earlier version assumed an additive specification and this alternative does not alter the results.

[46] This was not the case for monetary policy interacting with $disp_D$ shocks.

Figure 11: Monetary Responses



(a) Feedback to $disp_Q$ Shocks **(b)** Feedback to μ_Q Shocks

Note: This figure shows the effects of a response in monetary policy to $disp_Q$ and μ_Q shocks.

face as buyers. As in the previous analysis, all of the newly created money is distributed as a proportional transfer. But in this specification, it is feasible for the monetary authority to link these transfers to the current state of the economy. If prices were perfectly flexible, there would be no real effects of this monetary policy. Further, since private agents share the information of the monetary authority, there is no information transmitted to the private sector by this policy.

The SREE was characterized for both shocks to μ_Q and $disp_Q$. Consider first the results when the economy is driven by variations in $disp_Q$, along with money shocks. Figure 11a illustrates the outcome and Table 5 summarizes the moments.

In this case, the patterns in the simulated data match those in the actual data. A feedback rule with $\zeta = -0.05$ generates countercyclical dispersion in $disp_R$. With this policy, the monetary authority responds to higher than average dispersion in idiosyncratic profitability shocks by reducing the average growth of the money supply. In the absence of the intervention, output would be positively correlated with $disp_Q$. So, the monetary authority appears to be leaning against the wind. But in this case, the response to the policy outweighs the direct effect of $disp_Q$ so that increased dispersion in z is associated with an economic downturn. The dispersion in TFPR follows that of TFPQ, so that $disp_R$ is countercyclical. Note that this result does not occur without monetary feedback. As noted earlier, with $\zeta = 0$ the model creates procyclical dispersion in TFPR.

From Table 5, the frequency of price adjustment is higher in the recession. Further, the dispersion of price changes is also higher in recessions. Thus the frequency and dispersion of price changes positively comove. These patterns match those in the data. In terms of magnitudes, the variation in both the dispersion and frequency of price changes are close to those in the data but the variation in the dispersion of TFPR is much less.[47]

Another perspective on the magnitude of the variations uses Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) which reports that the dispersion in TFPR increases by about a factor of 4 between expansions and contractions. If the domain of $disp_Q$ is $[-0.5, 0.5]$, then, from Figure 11a, the variation in the standard deviation of TFPR is about 4. With this restriction, the variation in the frequency of price adjustment is larger than in the data.

A second interesting case arises from the response of the monetary authority to mu_Q shocks. This is illustrated in Figure 11b. In this setting, the nonlinear response of price setting to the state is important. Despite the monetary authority leaning against the wind, output increases with the mean of TFPQ. But, in contrast to the case with no monetary feedback in Figure 8a, now $disp_R$ varies considerably with the aggregate productivity shock. This is because of the response of price setters interacting with the money shock.

From Table 5, $disp_R$ is countercyclical, but not nearly as much as in the case of monetary feedback to variations in $disp_Q$. A difference with the data appear in the pricing moments. In particular, the frequency of adjustment is procyclical, produced by the asymmetry in the U-shaped hazard from Figure 8a.

In this model, the monetary authority responds directly to shocks rather than endogenous outcomes, such as an output gap. Of course, in equilibrium, the feedback can be generate correlations with, say, output. As reported, responding to $disp_Q$ with $\zeta < 0$ matches data patterns. In this case, the monetary authority was "leaning against the wind": with a countercyclical $disp_Q$, money growth is higher during economic contractions.

6 Extensions

This section contains four extensions of the baseline model.[48] The first considers an alternative parmeterization not based on micro moments. The second studies richer preferences that allow non-constant demand elasticity. The third looks at patterns of conditional correlations to highlight the inherent nonlinearities of the model. The final section introduces uncertainty.

6.1 Macro Calibration

Instead of parameterizing the model to match the moments based upon the steady state, the parameters are calibrated following Vavra (2014).[49] The moments matched are time series averages of pricing moments. The underlying model has shocks to the mean and dispersion of productivity as well as a shock to nominal spending. The parameterization is shown in Table 6. Key differences relative to the baseline is that in this

[47]Increasing the monetary feedback parameters so that money tightened more in response to an increase in $disp_Q$ did not produce larger variation in the dispersion of TFPR. This can be attributed to the nonlinearity inherent in the relationship between monetary shocks and TFPR dispersion as in Figure 6a, rendering the correlation between them nontrivial.

[48]We appreciate the guidance of referees in helping to design the first two of these extensions.

[49]Vavra (2014) sets $\sigma_d = 0$ as there are no idiosyncratic demand shocks in the macro calibration.

Table 6: Macro Parameterization

Parameter	Value	Description
Menu Cost Distribution		
ψ	0.09	Probability of zero menu cost
$\bar{F}$	0.09	Upper bound on menu cost
Utility Parameters		
ϵ	6.8	Elasticity of substitution between products
ϕ	2	Elasticity of labor supply
Idiosyncratic Productivity Shock		
σ_z	0.0378	Standard Deviation
Idiosyncratic Taste (Demand) Shock		
σ_d	0	Standard Deviation

parameteriation the elasticity of substitution between products is set at 6.8. Further, both the probability of a zero menu cost and the upper bound on the menu costs are higher than the initial specification.[50]

The micro moments from the steady state produced with this alternative calibration are shown in Table 7. Clearly the moments from the steady state with the micro calibration fit these moments better but the differences are not that large. The macro calibration creates a bit more dispersion of TFPR and also a bit more relative dispersion between TFPQ and TFPR. In terms of the frequency of price changes, the macro calibration brings the model closer to the data moments, partly due to the higher ε.

Table 7: Matching Moments: Calibrations

Moment	Data	Micro	Macro	Kimball Macro	Kimball Micro	Source
$disp_R$	0.102	0.103	0.114	0.113	0.107	Vavra (2014)
$disp_Q/disp_R$	1.181	1.181	1.187	1.071	1.131	Foster, Haltiwanger, and Syverson (2008)
$freq_{\Delta p}$	0.110	0.127	0.115	0.209	0.171	Vavra (2014)

Note: This table shows cross sectional basic moments computed from time series averages and the steady state of our model using macro calibrated parameters, shown in Table 6, and the Kimball aggregator. Here Kimball Macro has $\eta = -5$ and Kimball Micro has $\eta = -1.27$. All variables are logarithms except for frequency of price adjustment.

Table 8 summarizes the moments from the various experiments for the model with the macro parameters. Qualitatively, the results are very similar to the baseline parameterization. There are again two leading cases in which the data patterns of countercyclical TFPR variation are matched: (i) negatively correlated shocks to the mean and dispersion of TFPQ and (ii) leaning against the wind in response to shocks to the dispersion of TFPQ. As in the baseline results, for these two cases, the dispersion of TFPR, the dispersion of price changes and the frequency of adjustment are countercyclical. For these cases, the levels of dispersion in

[50]This is shown in Appendix Figure A.5b.

TFPR and price changes as well as the frequency of adjustment are much lower than in the baseline, as are the differences between expansions and contractions.

Table 8: Cyclical Variations with Macro Parameterization

Shock	$disp_R$		$disp_{\Delta p}$		$freq_{\Delta p}$	
	Contraction	Expansion	Contraction	Expansion	Contraction	Expansion
Data						
	0.219	0.051	0.090	0.073	0.161	0.149
Individual Shocks						
$disp_Q$	0.111	0.168	0.012	0.034	0.100	0.183
x	0.131	0.134	0.024	0.024	0.143	0.166
μ_Q	0.137	0.126	0.022	0.030	0.133	0.187
Correlated Shocks						
$disp_Q, \mu_Q$	0.180	0.116	0.039	0.016	0.209	0.105
Leaning Against the Wind						
$disp_Q$	0.135	0.121	0.028	0.022	0.176	0.152
μ_Q	0.134	0.127	0.028	0.031	0.171	0.332

Moments

Note: This table shows the cyclical patterns of the dispersion in TFPR, $disp_R$, the dispersion in price changes, $disp_{\Delta p}$ and the frequency of price adjustment, $freq_{\Delta p}$, using the macro calibration.

6.2 Kimball Aggregator

This section explores another factor that might impact pricing and thus TFPR: preferences with non-constant elasticity of substitution.[51] A prominent case is that of the Kimball aggregator, based upon Kimball (1995), explored in a number of applications, including Dotsey and King (2005), Smets and Wouters (2007), Klenow and Willis (2016) and Berger and Vavra (2019) and among others. The key is to maintain the tractability of the CES structure while providing some state dependence to the elasticity of substitution and thus price setting through a state dependent markup.[52] The consequent state dependence of markups can potentially lead to richer variations in the distribution of TFPR.

Specifically, we follow the specification in Dotsey and King (2005) which neatly captures the specification of a "kinked" demand curve such that demand is more responsive to relative price increases than decreases.[53] From their equation (2), demand for product i relative to the consumption aggregate can be expressed as:

$$\frac{c^i}{c} = \frac{1}{1+\eta}[\left((\frac{p^i}{P})(\frac{P}{\Lambda})\right)^{\frac{1}{(\gamma-1)}} + \eta] \tag{17}$$

[51] We appreciate the comments of a referee that led to the consideration of this case.

[52] An intriguing alternative would be through search, as explored in Qiu and Ríos-Rull (2022).

[53] We chose the Dotsey and King (2005) version to be close to their findings about the effects of money shocks. Harding, Lindé, and Trabandt (2022) and Aruoba, Oue, Saffie, and Willis (2023) also use this specification. Other papers, such as Berger and Vavra (2019), use the Klenow and Willis (2016) specification.

where Λ is a multiplier determined by the expenditure minimization problem underlying the aggregator. This simplifies to (1) when $\eta = 0$. For $\eta > 0$, the properties are neatly displayed in Figure 1 of Dotsey and King (2005) where: (i) marginal revenue is less sensitive to quantity above the kink and more sensitive below, (ii) the elasticity of demand is falling in the level consumed, and (iii) the profit function (over quantity) displays more curvature and is asymmetric, with profits falling rapidly with quantity above the kink. Put differently, the demand is much more elastic with respect to price increases compared to price reductions. Accordingly, an increase in marginal cost due to a decline in productivity will lead to a smaller price increase than a comparable reduction in marginal cost.

We present results for two calibrations.[54] They differ in the magnitude of η and thus the strength of the complementarity in the pricing decision. For one, termed the micro calibration, $\eta = -1.27, \gamma = 1.83$ following the calibration in Aruoba, Oue, Saffie, and Willis (2023) which matched the frequency of price changes, the fraction of price increases and the size of price changes based on the micro evidence presented in Foster, Haltiwanger, and Syverson (2008). For the other, termed the macro calibration, $\eta = -5, \gamma = 1.2$ allowing more complementarity.

For the analysis of TFPR, this alternative representation of preferences will matter for pricing decisions in a few ways. First, the kink in demand increases the incentives to adjust, thus impacting the extensive margin and the distribution of TFPR. From the steady state equilibrium, the adjustment rate is nearly 21% with the Kimball aggregator ($\eta = -5$) compared to 12.7 % in the CES baseline, as seen from the Kimball column of Table 7. Second, on the intensive margin the variable elasticity of demand implies a change in the responsiveness of prices to variations in idiosyncratic productivity, as well as other shocks. Further, with the added incentive to set individual prices close to the aggregate price, the dispersion in price changes (and thus prices) is quite small in the steady state: it is only 0.024 when $\eta = -5$ compared to the baseline case of 0.114.

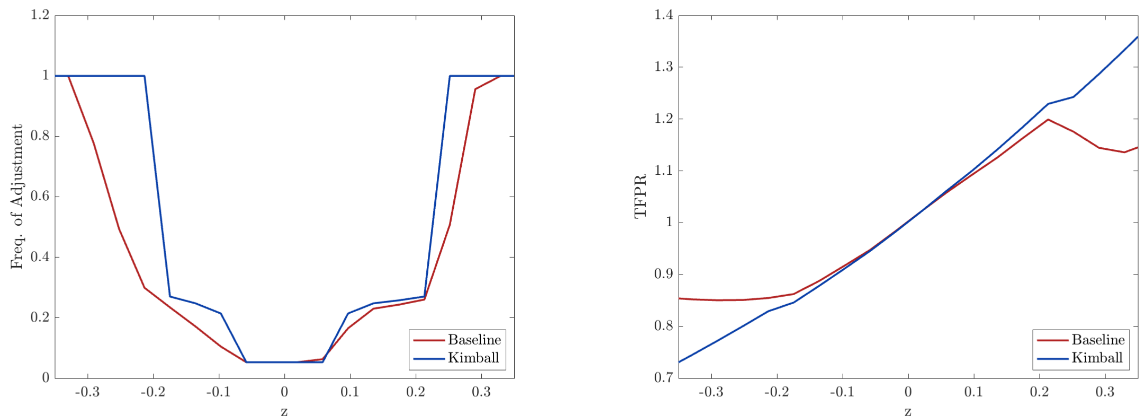
To illustrate, the left panel of Figure 12 shows the frequency of adjustment. An increase in the adjustment rate with the Kimball aggregator is evident as sellers pay a larger penalty from deviating from others. The right panel illustrates the relationship between firm level TFPR and productivity for each of the two aggregators. This mixes the adjustment margin and the sensitivity of the *ex post* price to z for adjustors.[55] Prices are less responsive to productivity under the Kimball aggregator so that TFPR is more responsive to z, as shown in the figure. As the elasticity of demand varies with the quantity sold, prices are higher under the Kimball aggregator when z is high and lower when productivity is below average.

Looking across the steady states, the dispersion of TFPR is slightly higher with the non-CES aggregator compared to the baseline. With $\eta = -5$ the TFPR dispersion is 0.107 while it is 0.103 in the baseline. This can be seen through (13) as both a reduction in the dispersion of prices and a fall in the (absolute value) covariance of prices and productivity.

[54] If we set $\eta = -12$ and $\gamma = 1.01$, close to the parameterization in both Dotsey and King (2005) Harding, Lindé, and Trabandt (2022), the dispersion of price changes is essentially zero reflecting the strong incentives to align prices. Thus we focus on parameterizations with lower absolute values of η.

[55] Thus TFPR is not necessarily monotone in z as it would be if we conditioned only on adjustors.

Figure 12: TFPR: Impact of Kimball Aggregator



(a) Frequency of Adjustment **(b)** Relating TFPR to z

Note: This figure compares the frequency of adjustment and the dependence of TFPR on productivity for CES and non-CES preferences, the macro calibration.

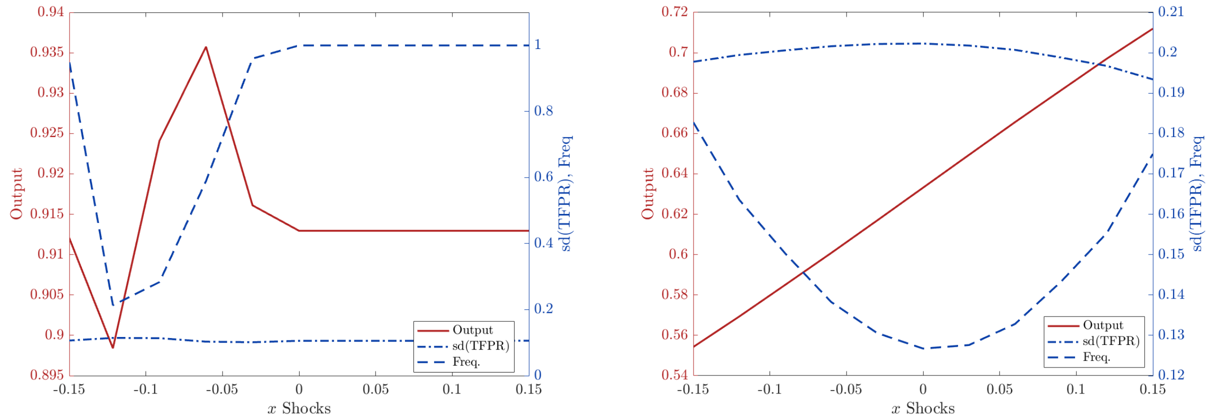
To understand the impact of these preferences on the dispersion of TFPR in our model, we return to the money shock experiment. Figure 13a displays output, the standard deviation of TFPR and the frequency of adjustment with $\eta = -5$. The results compare to the right panel of Figure 6, reproduced here in the right panel, where $\eta = 0$. As is evident from comparing the figures, the response of output to the money shock is considerably muted in the non-CES case: the range of money shocks displaying non-neutrality is quite narrow. This is because there is a relatively small range of shocks where the frequency of price adjustment is less than one. Further, there is essentially no variation in the dispersion of TFPR in this case. There are two other features of the model with non-CES preferences to note that arise from the asymmetries in the incentives to adjust prices. First, the *ex ante* price is different from that in the CES case so that the range of inaction shifts to the left.[56] Second, there is more sluggishness in price adjustment and thus a larger output response for low values of the money shocks compared to high realizations.[57]

The experiments for the two calibrations and various shocks are reported in Table 9. Here we see that the baseline results are robust to this alternative representation of preference, particularly for the lower value of η. In particular, the case that mixes shocks to the mean and dispersion of TFPQ again produce data consistent patterns of countercyclical dispersion in TFPR, dispersion in price changes and in the frequency of price changes. The same is true for the case in which the monetary authority leans against the wind in response to a shock to the dispersion of TFPQ. Note that due to the stronger strategic complementarity in pricing, the price adjustment frequencies are much higher than in the baseline.

[56] The *ex ante* price for the baseline CES is 1.773 and it falls to 1.117 when $\eta = -5$ for the Kimball aggregator.

[57] Harding, Lindé, and Trabandt (2022) also find a related asymmetry in response to shocks driven by differences in gains to price adjustment in expansions versus contractions.

Figure 13: Money Shock



(a) $\eta = -5$ **(b)** $\eta = 0$

Note: This figure shows the relationship between output, $disp_R$, the frequency of price adjustment and the price gap as a function of money shocks. There are two cases: panel a is for non-CES and panel (b) is for CES preferences.

6.3 Nonlinearities

As noted earlier, correlations have not been used to summarize model properties given the inherent nonlinearities created by the price adjustment hazard. Thus in our consideration of the various sources of fluctuations in $disp_R$, we have focused more on moments conditional on the state of aggregate activity, either contractions or expansions. The point of this section is to make clear that simple correlations can mask underlying nonlinearities in these state dependent pricing models.

To be clear, these nonlinearities are a direct consequence of the U-shaped hazard. As that is a central feature of state dependent pricing models, these properties are not peculiar to our specification. Given that, model and data statistics ought to be treated in a manner that is consistent with the inherent non-linearities of these economies. There are, in fact, two empirical studies, Ascari and Haber (2022) and Tenreyro and Thwaites (2016), that focus on nonlinear responses of output to monetary innovations.

One way to highlight the importance of this is to compute correlations conditional on the business cycle, measured by the difference between aggregate output and its steady state value. Table 10 presents correlations of key variables with output conditional on whether output is above (expansion) or below (contraction) its stationary level. This is shown for the various sources of fluctuations, including cases with the monetary feedback rules. The fact that these correlations are state independent reflects both the nonlinear decision rules and that the simulated distributions put non-negligible weight on these areas.

First, looking at the monetary shock case, the frequency of price adjustment is negatively correlated with output in a contraction and positively correlated with output in an expansion, -0.799 and 0.830 respectively. This is a direct consequence of the U-shaped hazard, as in Figure 6a. So when the money shock is above

Table 9: Cyclical Variations with Kimball Aggregator

Shock	Moments					
	$disp_R$		$disp_{\Delta p}$		$freq_{\Delta p}$	
	Contraction	Expansion	Contraction	Expansion	Contraction	Expansion
			Micro Calibration ($\eta = -1.27$)			
			Individual Shocks			
$disp_Q$	0.102	0.104	0.056	0.063	0.211	0.231
x	0.100	0.090	0.045	0.035	0.262	0.729
μ_Q	0.097	0.102	0.032	0.049	0.371	0.246
			Correlated Shocks			
$disp_Q, \mu_Q$	0.117	0.097	0.061	0.039	0.240	0.169
			Leaning Against the Wind			
$disp_Q$	0.103	0.094	0.044	0.035	0.484	0.478
μ_Q	0.099	0.094	0.040	0.040	0.321	0.533
			Macro Calibration ($\eta = -5$)			
			Individual Shocks			
$disp_Q$	0.116	0.112	0.023	0.024	0.236	0.210
x	0.103	0.104	0.016	0.013	0.978	0.494
μ_Q	0.113	0.120	0.046	0.037	0.211	0.290
			Correlated Shocks			
$disp_Q, \mu_Q$	0.485	0.428	0.025	0.020	0.107	0.071
			Leaning Against the Wind			
$disp_Q$	0.128	0.083	0.019	0.012	0.936	0.825
μ_Q	0.104	0.105	0.016	0.016	0.981	0.630

Note:This table shows the cyclical patterns of the dispersion in TFPR, $disp_R$, the dispersion in price changes, $disp_{\Delta p}$ and the frequency of price adjustment, $freq_{\Delta p}$. The moments are displayed as columns, for contractions and expansions.

average, so is output. Within this region, higher realizations of the money shock increase the frequency of price adjustment and, at the same time, output expands. But, for values of the money shock below the mean (so that output is below its mean), the opposite occurs. For progressively lower values of x, again the frequency of price adjustment rises but output falls, producing a negative correlation in this region. The unconditional correlation is negative. It masks the positive comovement between output and the frequency of price adjustment in expansionary periods.

The dispersion of TFPR has an inverted U shape in Figure 6a. This produces a negative correlation with output in expansions of -0.727 when x is above its mean. But the correlation switches sign when x is below its mean. Again, this is not captured by the unconditional correlation.

Second, note that in many cases other than money shocks, the correlations change sign with the state of the economy. This pattern of a positive (negative) correlation of price adjustment and output in expansions

Table 10: Cyclical Variations: Conditional Correlations with Output

Shock	$disp_R$			$disp_{\Delta p}$			$freq_{\Delta}p$		
	Unc.	Cont.	Exp.	Unc.	Cont.	Exp.	Unc.	Cont.	Exp.
				Individual Shocks					
x	0.499	0.853	-0.727	-0.044	-0.896	0.911	-0.216	-0.799	0.830
$disp_Q$	0.721	0.322	0.952	0.893	0.787	0.952	0.911	0.799	0.842
$disp_D$	0.023	0.028	-0.034	0.403	0.122	0.337	0.032	-0.066	0.131
μ_Q	0.076	0.241	-0.167	-0.118	-0.883	0.911	0.068	-0.613	0.663
				Correlated Shocks					
$disp_Q, \mu_Q$	-0.812	-0.740	-0.815	-0.906	-0.902	-0.965	-0.810	-0.961	-0.800
				Leaning Against the Wind					
$disp_Q$	-0.074	0.664	-0.246	-0.091	0.043	0.097	-0.169	-0.137	0.147
μ_Q	0.047	-0.277	-0.155	-0.022	0.319	0.205	-0.028	0.254	0.189

Note: This table shows the conditional correlation with output of the dispersion in TFPR, $disp_R$, the dispersion in price changes, $disp_{\Delta p}$ and the frequency of price adjustment, $freq_{\Delta p}$. Here contractions (Cont.) and expansions (Exp) are defined in levels relative to steady state and Unc. is the unconditional correlation.

(contractions) is seen in the other cases except for $disp_Q$ shocks. In that case, the frequency of price adjustment is higher in expansions but, from Table 10, the correlation with output is negative, conditional on being in an expansion.

Third, variations in shocks to x, $disp_D$ or μ_Q can each produce countercyclical $disp_R$ but only during expansions. The combination of $disp_Q, \mu_Q$ shocks generate this negative correlation in all states of the business cycle.

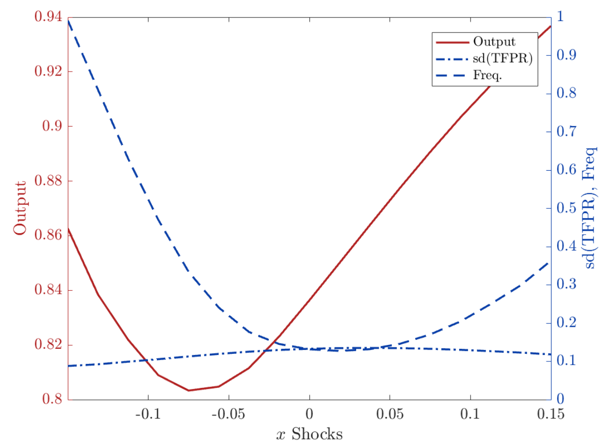
There is an interesting form of nonlinearity brought out in the response to monetary shocks in the macro parameterization.[58] As seen in Figure 14, there is a region in which the correlation of output and the money shock becomes negative. That is, related to the non-neutrality of money is a non-linearity in the response of output to money shocks. For this parameterization, for extremely low realizations of x, output will be independent of x since all sellers will adjust their prices. At the same time, for intermediate values of x, output is increasing in the money shock. In order for these two regions to coexist, the correlation of output and the money shock must switch sign, with output decreasing in x over a region as shown in the figure. For this case there is a wide region in which the output is positively correlated with the dispersion in TFPR. As in the baseline model, the dispersion rises for extreme values of the money shock.

6.4 Effects of Uncertainty

The distinction between uncertainty and dispersion is often blurred. The main effect of uncertainty, again expressed in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), is to create an incentive to wait and

[58] This is a general property of state dependent pricing models responding to monetary shocks for a large enough domain. It is clear here as we do not linearize around a steady state equilibrium.

Figure 14: Money Shocks: Macro Calibration



Note: This figure shows the effects of money shocks for the macro calibration.

allow the uncertainty to be resolved before making an irreversible choice, such as changing a price. To the extent this leads to a decrease in spending, largely on durables, the uncertainty can be recessionary. This is often quite different from the positive effects of dispersion which can lead to an expansion in output, as discussed above.

The previous discussion highlighted the effects of dispersion on the frequency of price adjustment and thus the real effects of monetary shocks. Here we focus on how *ex ante* prices respond to uncertainty over a distribution, not the realization of that change.

Our analysis includes distributions over four dimensions: (i) idiosyncratic productivity, (ii) idiosyncratic demand, (iii) money shocks, and (iv) aggregate productivity. Thus in principle one can study the effects of uncertainty with respect to each of these four distributions.

To do so, it is natural to create a Markov switching process for the dispersion of, say, idiosyncratic productivity. Price setters in period t would know the distribution of these shocks last period but in setting their *ex ante* price, the period t distribution, as well as that for period $t+1$ would not be known. Further, for those who adjust *ex post*, the uncertainty would remain over the distribution in the following period when they are consumers.[59] This is the nature of the uncertainty.

One extreme version of this Markow switching process is for the dispersion to be permanently high (low). For the price setting problem of young agents, the *ex ante* price is essentially the same with high dispersion of the idiosyncratic productivity shock as it is for the low dispersion case. In fact, this is true when the uncertainty is over the money transfer or the aggregate productivity distributions.

Given this, it is unlikely that *ex ante* uncertainty matters for the price setting problem. This is verified explicitly for the case of uncertainty over idiosyncratic productivity. Even if there is a positive probability of a regime shift in the distribution of z, the *ex ante* price is essentially unchanged.

[59] Thus the expectation on the left side of (A.8) is extended to include the conditional expectation over the future dispersion.

This is an important finding. It makes clear that the effects come from dispersion not uncertainty. This is consistent with Berger, Dew-Becker, and Giglio (2020) who argue, at least for aggregate shocks, that uncertainty *per se*, had a negligible effect on real activity.

7 Conclusion

The analysis characterizes the properties of the distribution of TFPR in a stationary rational expectations equilibrium of a monetary economy with state dependent pricing. A quantitative version of the model is used to determine the cyclicality of the dispersion in TFPR as well as other key pricing moments, the cyclicality of both the frequency of price changes and their dispersion. This is studied by determining pricing decisions and thus the distribution of TFPR in the face of aggregate shocks to: (i) the money supply, (ii) the dispersion of TFPQ, (iii) the mean of TFPQ and (iv) the dispersion of demand. These are very conventional shocks for an aggregate economy, with recent attention given to variations in the dispersion of TFPQ and demand.

The moments are generated from a stationary rational expectations equilibrium without the need for linearization. This matters as the firm-level non-linearities in the state dependent pricing model carry over to the aggregate economy.

Looking at these shocks alone as well as combinations and allowing monetary feedback, there are a few cases in which the data patterns of countercyclicality in the dispersion of TFPR, the frequency of price adjustment and dispersion in price changes are matched. One case arises when there are negatively correlated shocks to the mean and dispersion of TFPQ. This combination was highlighted in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) to match aggregate fluctuations. Here the combination actually creates the countercyclical dispersion in TFPR assumed in that paper. Also, a monetary authority that leans against the wind in face of shocks to the dispersion of TFPQ creates an equilibrium that matches data patterns.

Finally, the model is used to study the effects of uncertainty on pricing. It seems clear that the effects highlighted in our analysis stem from dispersion not uncertainty. One interesting extension of our model would be to include some of the adjustment cost structure that creates a real options effect, as in Bloom (2009), **coupled with** state dependent pricing.

Admittedly these results are suggestive rather than definitive. The OG model, with only one period of price setting, misses some of the forward looking aspect of price adjustment. But, as argued in the text, the pricing behaviour in the model is similar to that produced by other state dependent pricing models. Nonetheless, extending the model to include additional state variables, say through habit formation of households or some form of customer capital, would be of interest.[60]

Throughout these exercises, one theme emerges: non-linearities in the response of the economy to monetary and dispersion shocks. Regardless of the source of aggregate fluctuations, the dispersion of TFPR is generally lowest for extremely low and high realizations and highest for the average state. This property of

[60]As suggested by a referee, this would allow us to connect to the additional evidence in Foster, Haltiwanger, and Syverson (2016).

the model, driven by the U-shaped response of the frequency of price changes to money surprises, makes it useful to study the impact of monetary and productivity shocks using non-linear statistical models. For this, there is clear value in looking further at price adjustment frequency as well as employment and output responses, at both the firm and aggregate levels, in a non-linear setting.

The model has unexplored implications for the distribution of markups. As discussed in the literature on markups, for example in Nekarda and Ramey (2020), being explicit about the source of fluctuations is important for studying the cyclicality of markups. Our intention is to go further to study the cyclicality of the distribution of markups.

References

Abel, A. B. (1983): "Optimal investment under uncertainty," The American Economic Review, 73(1), 228–233.

Aruoba, S. B., E. Oue, F. Saffie, and J. L. Willis (2023): "Real Rigidities, Firm Dynamics, and Monetary Nonneutrality: The Role of Demand Shocks," Discussion paper, Federal Reserve Bank of Atlanta, Working Paper 2023-3.

Ascari, G., and T. Haber (2022): "Non-linearities, state-dependent prices and the transmission mechanism of monetary policy," The Economic Journal, 132(641), 37–57.

Asker, J., A. Collard-Wexler, and J. De Loecker (2014): "Dynamic inputs and resource (mis) allocation," Journal of Political Economy, 122(5), 1013–1063.

Bachmann, R., and C. Bayer (2014): "Investment Dispersion and the Business Cycle," American Economic Review, 104(4), 1392–1416.

Bachmann, R., B. Born, S. Elstner, and C. Grimme (2019): "Time-varying business volatility and the price setting of firms," Journal of Monetary Economics, 101, 82–99.

Ball, L., and D. Romer (1991): "Sticky Prices as Coordination Failure," The American Economic Review, 81(3), 539.

Berger, D., I. Dew-Becker, and S. Giglio (2020): "Uncertainty shocks as second-moment news shocks," The Review of Economic Studies, 87(1), 40–76.

Berger, D., and J. Vavra (2019): "Shocks versus responsiveness: What drives time-varying dispersion?," Journal of Political Economy, 127(5), 2104–2142.

Bloom, N. (2009): "The Impact of Uncertainty Shocks," Econometrica, 77(3), 623–685.

Bloom, N., M. Floetotto, N. Jaimovich, I. S. Eksten, and S. J. Terry (2018): "Really Uncertain Business Cycles," Econometrica, 86(3), 1031–1065.

Caballero, R. J., and E. M. Engel (1993): "Heterogeneity and output fluctuations in a dynamic menu-cost economy," The Review of Economic Studies, 60(1), 95–119.

——— (2007): "Price stickiness in Ss models: New interpretations of old results," Journal of monetary economics, 54, 100–121.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal rigidities and the dynamic effects of a shock to monetary policy," Journal of political Economy, 113(1), 1–45.

Decker, R. A., J. C. Haltiwanger, R. S. Jarmin, and J. Miranda (2019): "Changing business dynamism and productivity: Shocks vs. responsiveness," Discussion paper, National Bureau of Economic Research.

Dew-Becker, I., and S. Giglio (2023): "Cross-sectional uncertainty and the business cycle: evidence from 40 years of options data," American Economic Journal: Macroeconomics, 15(2), 65–96.

Dotsey, M., and R. G. King (2005): "Implications of state-dependent pricing for dynamic macroeconomic models," Journal of Monetary Economics, 52(1), 213–242.

Dotsey, M., R. G. King, and A. L. Wolman (1999): "State-dependent pricing and the general equilibrium dynamics of money and output," The Quarterly Journal of Economics, 114(2), 655–690.

Dotsey, M., and A. L. Wolman (2019): "Investigating Nonneutrality in a State-Dependent Pricing Model with Firm-Level Productivity Shocks," FRB of Philadelphia Working Paper.

Eslava, M., and J. C. Haltiwanger (2020): "The Life-Cycle Growth of Plants: The Role of Productivity, Demand and Wedges," Discussion paper, National Bureau of Economic Research.

Foster, L., J. Haltiwanger, and C. Syverson (2008): "Reallocation, firm turnover, and efficiency: Selection on productivity or profitability?," American Economic Review, 98(1), 394–425.

——— (2016): "The slow growth of new plants: Learning about demand?," Economica, 83(329), 91–129.

Galí, J. (2015): Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications. Princeton University Press.

Garcia-Marin, A., and N. Voigtländer (2019): "Exporting and plant-level efficiency gains: It's in the measure," Journal of Political Economy, 127(4), 1777–1825.

Golosov, M., and R. E. Lucas (2007): "Menu costs and Phillips curves," Journal of Political Economy, 115(2), 171–199.

Harding, M., J. Lindé, and M. Trabandt (2022): "Resolving the missing deflation puzzle," Journal of Monetary Economics, 126, 15–34.

Hartman, R. (1972): "The effects of price and cost uncertainty on investment," Journal of economic theory, 5(2), 258–266.

Hottman, C. J., S. J. Redding, and D. E. Weinstein (2016): "Quantifying the sources of firm heterogeneity," The Quarterly Journal of Economics, 131(3), 1291–1364.

Hsieh, C.-T., and P. J. Klenow (2009): "Misallocation and manufacturing TFP in China and India," The Quarterly Journal of Economics, 124(4), 1403–1448.

Kehrig, M. (2011): "The cyclicality of productivity dispersion," US Census Bureau Center for Economic Studies Paper No. CES-WP-11-15.

Kimball, M. D. (1995): "The quantitative analytics of the basic neomonetarist model," Journal of Money, Credit, and Banking, 27(4), 1241.

Klenow, P. J., and B. A. Malin (2010): "Microeconomic evidence on price-setting," in Handbook of monetary economics, vol. 3, pp. 231–284. Elsevier.

Klenow, P. J., and J. L. Willis (2016): "Real rigidities and nominal price changes," Economica, 83(331), 443–472.

Krusell, P., and A. A. Smith, Jr (1998): "Income and wealth heterogeneity in the macroeconomy," Journal of political Economy, 106(5), 867–896.

Lucas, R. E. (1972): "Expectations and the Neutrality of Money," Journal of economic theory, 4(2), 103–124.

Nekarda, C. J., and V. A. Ramey (2020): "The cyclical behavior of the price-cost markup," Journal of Money, Credit and Banking, 52(S2), 319–353.

Oi, W. Y. (1961): "The desirability of price instability under perfect competition," Econometrica: journal of the Econometric Society, pp. 58–64.

Qiu, Z., and J.-V. Ríos-Rull (2022): "Procyclical productivity in new keynesian models," Discussion paper, National Bureau of Economic Research.

Sedlacek, P., and M. Ignaszak (2021): "Productivity, Profitability and Growth," CEPR Discussion Paper No. DP16205.

Smets, F., and R. Wouters (2007): "Shocks and frictions in US business cycles: A Bayesian DSGE approach," American economic review, 97(3), 586–606.

Tenreyro, S., and G. Thwaites (2016): "Pushing on a string: US monetary policy is less powerful in recessions," American Economic Journal: Macroeconomics, 8(4), 43–74.

Vavra, J. (2014): "Inflation dynamics and time-varying volatility: New evidence and an SS interpretation," The Quarterly Journal of Economics, 129(1), 215–258.

A Online Appendix

A.1 Idiosyncratic Demand Shocks

In the presence of idiosyncratic demand shocks, consider a consumption aggregator of

$$c = \left(\sum_i \alpha_i c_i^{\gamma} \right)^{\frac{1}{\gamma}}$$

with $\gamma \equiv \frac{\varepsilon - 1}{\varepsilon}$. In this specification, α_i is a weight on good i as not all goods are weighted equally in utility. Relative demands are given by

$$\frac{c_i}{c_j} = \left(\frac{\alpha_j p_i}{\alpha_i p_j} \right)^{-\sigma}$$

where $\sigma = \frac{1}{\gamma - 1}$. Define $\tilde{P} = (\sum_i \tilde{p}_i^{1-\sigma} \alpha_i)^{1/(1-\sigma)}$, so that

$$c_j = (\frac{\tilde{p}_j}{\tilde{P}})^{-\sigma} \frac{M}{\tilde{P}} = \alpha_j^{\sigma} (\frac{p_j}{\tilde{P}})^{-\sigma} \frac{M}{\tilde{P}}. \tag{A.1}$$

Here M is nominal spending and $\tilde{p_j} \equiv \frac{p_j}{\alpha_j}$.

We introduce relative demand shocks through this specification. For a given distribution of weights, there is nothing stochastic about the household problem with respect to tastes. That is, the young agent of generation t has fixed preferences of consumption goods when they are old. So the household problem specified in the main text remains, with the modified aggregator.

But, from the perspective of a seller, the model introduces uncertainty in that *ex ante* the seller does not know the taste shock pertaining to the particular good of that seller. This allows uncertainty in demand to exist from the perspective of a seller but not the consumer. The uncertain demand can impact the *ex ante* price as well as the *ex post* decision to adjust and, conditional on adjustment, the *ex post* price.

This specification leads to two types of shocks. First, there are seller specific realizations of demand shocks, denoted α, which directly impact revenues. Second there are variations in the distribution of α are studied through mean preserving spreads, denoted $disp_D$.

A.2 Generalized Definition of SREE

Here the definition of a stationary rational expectations equilibrium is generalized to include shocks to the distribution of plant-level productivity through μ_Q and $disp_Q$ as well as shocks to idiosyncratic demand, α and the distribution of the demand shocks, $disp_D$. Let $S = (x, \mu_Q, disp_Q, disp_D)$ be the aggregate state and $s = (z, \alpha, F)$ be the idiosyncratic state.[61] As earlier, M is the previous money stock and thus is known at the time prices are chosen *ex ante*.

A SREE is a set of price functions $(\bar{p}(M), \tilde{p}(M, S, s), P(M, S))$, value functions $(W^n(M, S, s), W^a(M, S, s))$,

[61] So here the notation is different from that in the text to be more explicit about aggregate and idiosyncratic variables.

and a critical value of the price adjustment cost, $F^*(M,S,s)$ satisfying: (i) individual optimization by young price setters and old consumers, (ii) market clearing and (iii) consistency of beliefs and expectations for all states. These conditions can be written:

- $\bar{p}(M)$ solves the *ex ante* pricing problem given the state dependent price index $P(M,S)$;

$$\bar{p}(M) = argmax_p E_{S,s,S'} \left\{ V\left(\frac{R\,(M;p,\alpha;P(M,S),x)\,x'}{P(Mx,S')} \right) - g\left(\frac{d(M;p,\alpha;P(M,S),x)}{\mu_Q z} \right) \right\} \tag{A.2}$$

  for all M.

- $\tilde{p}(M,S,s)$ solves the *ex post* pricing problem:

$$\tilde{p}(M,S,s) = argmax_p E_{S'} \left\{ V\left(\frac{R(M;p,\alpha;P(M,S),x)x'}{P(Mx,S')} \right) \right\} - g\left(\frac{d(M;p,\alpha;P(M,S),x)}{\mu_Q z} \right). \tag{A.3}$$

  given $P(M,S)$, for all (M,S,s);

- At the critical adjustment cost, $F^*(M,S,s)$, the seller is just indifferent between adjusting and not:

$$F^*(M,S,s) \equiv W^n(M,S,s) - W^a(M,S,s)$$

  for all (M,S,s), with $W^a(M,S,s)$ given by:

$$W^a(M,S,s) = E_{S'} \left\{ V\left(\frac{R(M;\tilde{p}(M,S,s),\alpha;P(M,S),x)x'}{P(Mx,S')} \right) \right\} - g\left(\frac{d(M;\tilde{p}(M,S,s),\alpha;P(M,S),x)}{\mu_Q z} \right), \tag{A.4}$$

  and $W^n(M,S,s)$ given by

$$W^n(M,S,s) = E_{S'} \left\{ V\left(\frac{R(M;\bar{p}(M),\alpha;P(M,S),x)x'}{P(Mx,S')} \right) \right\} - g\left(\frac{d(M;\bar{p}(M),\alpha;P(M,S),x)}{\mu_Q z} \right). \tag{A.5}$$

- $P(M,S)$ is the aggregate price index in state (M,S) given by:

$$P(M,S) = [E_s(1-\Omega(F^*(M,S,s)))\bar{p}(M)^{1-\varepsilon} + E_s(\Omega(F^*(M,S,s))\tilde{p}(M,S,s)^{1-\varepsilon})]^{\frac{1}{1-\varepsilon}} \tag{A.6}$$

where $d(M;\bar{p}(M),\alpha;P(M,S),x) = \alpha^{\varepsilon}\left(\frac{\bar{p}(M)}{P(M,S)}\right)^{-\varepsilon} Y$ and $d\,(M;\tilde{p}(M,S,s),\alpha;P(M,S),x) = \alpha^{\varepsilon}\left(\frac{\tilde{p}(M,S,s)}{P(M,S)}\right)^{-\varepsilon} Y$. Here $Y = \frac{Mx}{P(M,S)}$ is the equilibrium determined real value of money holdings.

A.3 SREE: Linear Quadratic

For the case of linear quadratic preferences, the SREE defined in section 2.3 becomes a set of functions $\{\bar{p}(M), \tilde{p}(M; z, \alpha; x, \mu_Q), F^*(M; z, \alpha; x, disp_Q, disp_D, \mu_Q), P(M; x, disp_Q, disp_D, \mu_Q)\}$ such that:

- $\bar{p}(M)$ solves the *ex ante* pricing problem given the state dependent price index $P(M; x, disp_Q, disp_D, \mu_Q)$;

$$\begin{aligned} \hat{\varepsilon}\, \bar{p}(M)\, E_{\alpha;x,x',disp_Q',disp_D',\mu_Q'} \left[\frac{x'}{P\left(Mx; x', disp_Q', disp_D', \mu_Q'\right)} d\big(M; \bar{p}(M), \alpha; x\big) \right] = \\ E_{z,\alpha;x,\mu_Q} \left[\frac{d\big(M; \bar{p}(M), \alpha; x\big)}{\mu_Q z} \right]^2 . \end{aligned} \tag{A.7}$$

- $\tilde{p}(M; z, \alpha; x, \mu_Q)$ solves the *ex post* pricing problem given the state dependent price index $P(M; x, disp_Q, disp_D, \mu_Q)$;

$$\begin{aligned} \hat{\varepsilon}\, \tilde{p}(M; z, \alpha; x, \mu_Q)\ E_{x',disp_Q',disp_D',\mu_Q'} \left[\tfrac{x'}{P(Mx; x', disp_Q', disp_D', \mu_Q')} \right] = \\ \tfrac{d\left(M; \tilde{p}(M; z, \alpha; x, \mu_Q), \alpha; x\right)}{{\mu_Q}^2 z^2} . \end{aligned} \tag{A.8}$$

- At the critical adjustment cost $F^*(M; z, \alpha; x, \mu_Q)$, the seller is just indifferent between adjusting and not:

  $F^*(M; z, \alpha; x, \mu_Q) = W^a(M; z, \alpha; x, \mu_Q) - W^n(M; z, \alpha; x, \mu_Q)$

- $P(M; x, disp_Q, disp_D, \mu_Q)$ is the aggregate price function in state $(M; x, disp_Q, disp_D, \mu_Q)$ given by:

$$\begin{aligned} P(M; x, disp_Q, disp_D, \mu_Q) = \\ \left[E_{z,\alpha}(1 - \Omega(F^*(M; z, \alpha; x, \mu_Q)))\bar{p}(M)^{(1-\epsilon)} + E_{z,\alpha}\Omega(F^*(M; z, \alpha; x, \mu_Q))\tilde{p}(M; z, \alpha; x, \mu_Q)^{(1-\epsilon)} \right]^{\frac{1}{1-\epsilon}} \end{aligned} \tag{A.9}$$

Throughout, $d\big(M; \bar{p}(M), \alpha; P(M; x, disp_Q, disp_D, \mu_Q), x\big) = \alpha^{\varepsilon} \left(\frac{\bar{p}(M)}{P(M; x, disp_Q, disp_D, \mu_Q)} \right)^{-\varepsilon} Y$ and $d\big(M; \tilde{p}(M; z, \alpha; P(M; x, disp_Q, disp_D, \mu_Q)\, x, \mu_Q), \alpha; x\big) = \alpha^{\varepsilon} \left(\frac{\tilde{p}(M; z, \alpha; x, \mu_Q)}{P(M; x, disp_Q, disp_D, \mu_Q)} \right)^{-\varepsilon} Y$. Here, note that $Y = \frac{Mx}{P(M; x, disp_Q, disp_D, \mu_Q)}$ is the real output and thus real spending.

A.4 Quantitative Approach

In this section, we outline the quantitative methodology employed in our analysis. We begin with an overview of simulation details and address the characterization of both idiosyncratic and aggregate shocks, as well as the structural dynamics of the economy. Subsequently, we elaborate on the parameterizations of menu costs utilized in our analyses. The section concludes with a description of the computational algorithm employed.

A.4.1 Simulation Details

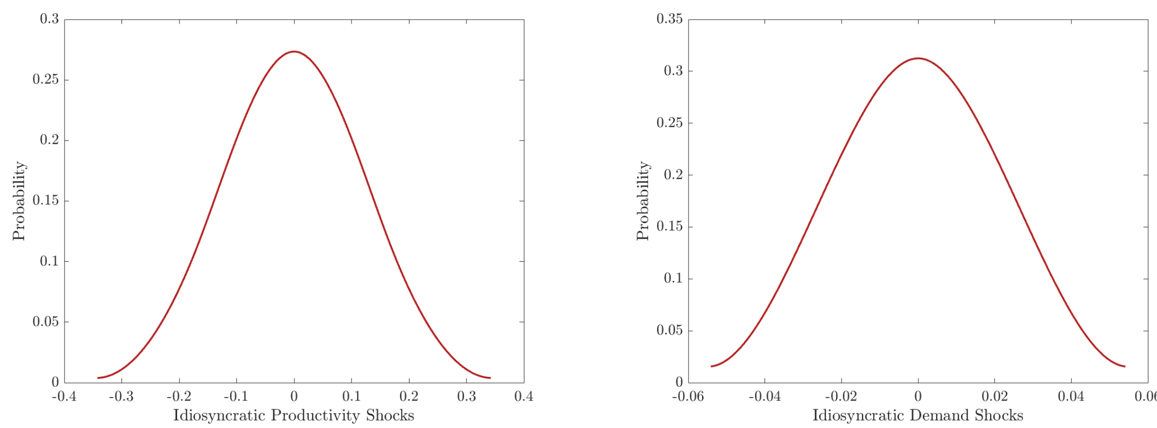
Within our simulation framework, we assume that all shocks follow a log-normal distribution. Further elaboration on the detailed parameters characterizing each shock is provided in the subsequent section. Given the relatively low probability of extreme realizations, it is unlikely that our results are significantly influenced by outliers.[62]

Model period is monthly. The simulated economy comprises $10,000$ firms, with the simulation spanning 1000 periods. We maintain a substantial number of firms and periods to thoroughly explore every point in the state space with our algorithm. This approach ensures that our results remain robust, unaffected by any random assignment of shocks.

A.4.2 Shocks

Idiosyncratic Shocks In the model, heterogeneity primarily arises from two sources: idiosyncratic productivity shocks (perturbations to TFPQ) and idiosyncratic demand shocks (unanticipated shifts in the demand for the goods produced by the firm). Under the baseline scenario *(without any dispersion shocks)*, firms possess perfect knowledge regarding the distribution from which they draw these shocks.

Figure A.1: Idiosyncratic Shocks



(a) z Shocks **(b)** α Shocks

Note: This figure shows the distribution of the log of the firm-level shocks: panel (a) is for productivity shocks and panel (b) is for demand shocks.

Both shocks follow a lognormal distribution. For idiosyncratic productivity shocks (in logs), characterized by a zero mean and standard deviation denoted by σ_z, we utilize the Rouwenhorst algorithm to generate a Markov matrix corresponding to a specified value of σ_z and mean z. The shape of the distribution for idiosyncratic productivity shocks is depicted in Figure A.1a. Similarly, idiosyncratic demand shocks exhibit

[62] We acknowledge the suggestion from a referee to illustrate where the model economy predominantly operates within the state space, thereby ensuring that our main results are not driven by outliers.

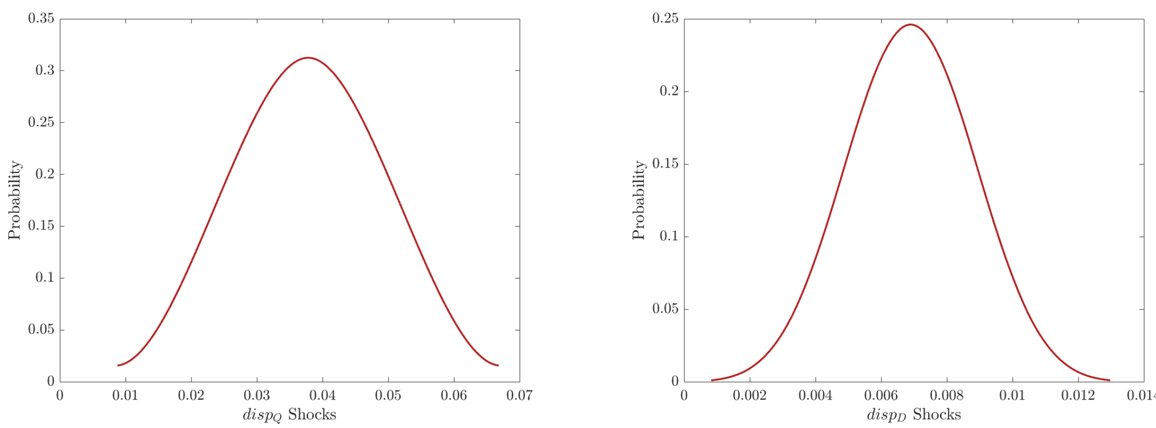
a zero mean and standard deviation, denoted by σ_d, with the distribution illustrated in Figure A.1b.

The standard deviations, σ_z and σ_d, are subject to variations induced by the $disp_Q$ and $disp_D$ shocks. In the absence of these shocks, the standard deviations assume values as depicted in Table 2, calibrated to align with empirical data moments.

Dispersion Shocks As mentioned earlier, our algorithm possesses the flexibility to relax the assumption that agents have complete information of the distribution from which they draw.

Upon the imposition of $disp_Q$ shocks, the spread in the idiosyncratic productivity distribution becomes a stochastic process. These shocks alter the dispersion of idiosyncratic productivity shocks for all agents, thus constituting an aggregate shock. Before the realization of $disp_Q$ shocks, agents *ex ante* do not know whether they will draw their idiosyncratic productivity from a wider or narrower distribution. In the *ex post* world, after realization of shocks in the economy, the realized value of $disp_Q$ sets the standard deviation of idiosyncratic shock distribution for all agents in the economy.

Figure A.2: Dispersion Shocks



(a) $disp_Q$ Shocks **(b)** $disp_D$ Shocks

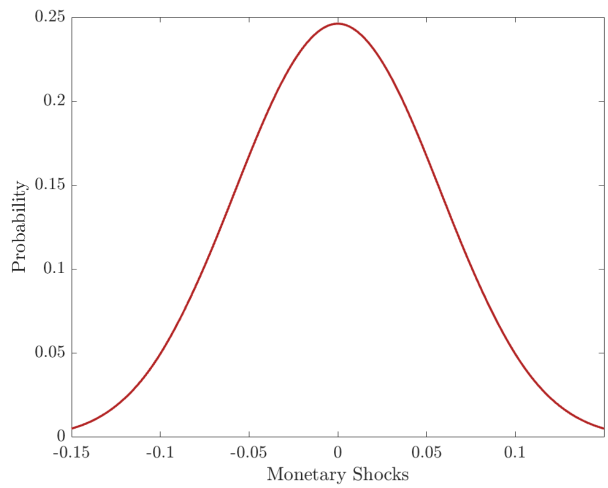
Note: This figure shows the distribution of the log of the dispersion shocks: panel (a) is for $disp_Q$ shocks and panel (b) is for $disp_D$ shocks.

The $disp_Q$ shocks follow a lognormal distribution, featuring 11 points on the grid. The ratio of the 9th value to the 3rd value is 3.94, slightly differing from the 4.1 ratio observed in Bloom et al. (2018). This distribution's shape is visualized in Figure A.2a.

Similarly, $disp_D$ shocks govern the dispersion of the idiosyncratic demand shock distribution, also following a lognormal distribution. The mean value of $disp_D$ shock is determined through calibration and is relatively small. The grid for $disp_D$ ranges from 0 to 0.011, with the mean value derived from calibration. The distribution's shape is depicted in Figure A.2b.

Monetary Shocks The distribution of x shocks depicted in Figure A.3 exhibits a bell-shaped curve centered on a mean of zero growth. While the figure highlights the presence of states with large money growth, their occurrence is highly infrequent. This suggests that our results are unlikely to be driven by such outliers, given their low probability of realization.

Figure A.3: Monetary Shock Distribution



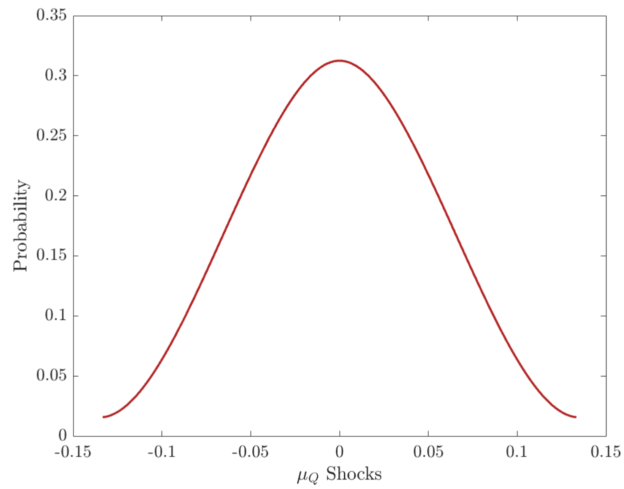
Note: This figure shows the distribution of money growth shocks.

μ_Q **Shocks** The imposition of μ_Q shocks transforms the mean of the z distribution into a stochastic process. In the *ex ante* world, firms are unaware of the mean of their idiosyncratic productivity shock distribution due to these shocks. In the *ex post* world, the realization of μ_Q shocks dictates the mean of the idiosyncratic productivity shock distribution, analogous to the effects of $disp_Q$ shocks. It is important to note that, in relation to the x shock scenario, due to the log-normal distribution of μ_Q shocks, the probability of extreme realizations is considerably low.

A.4.3 Menu Costs

Baseline Firm heterogeneity in our model is attributed not only to the variations in idiosyncratic productivity and demand shocks but also in firm-specific price adjustment costs. A segment of firms, quantified by a small fraction ψ, incurs no price adjustment costs. The majority of firms, represented by the fraction $1-\psi$, are subject to a nondegenerate and diverse distribution of adjustment costs. Therefore, the parameter ψ is pivotal in dictating the proportion of firms with flexible pricing; and thus an increase in ψ translates into a rise in the incidence of minor price adjustments and an overall higher frequency of price changes. By following Dotsey and Wolman (2019), a tangent function governs the menu cost distribution which is depicted in Figure A.5a.

Figure A.4: μ_Q Shock Distribution



Note: This figure shows the distribution of μ_Q shocks.

$$G(F) = \frac{1}{\omega}\left\{\tan(\frac{F-\kappa_2}{\kappa_1}) + \nu \cdot \pi\right\} \tag{A.10}$$

where the parameters are defined as

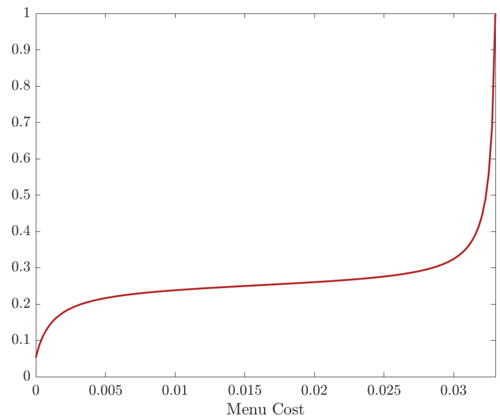
$$\kappa_1 = \frac{\bar{F}}{[\tan^{-1}(\omega - \nu \cdot \pi) + \tan^{-1}(\nu \cdot \pi)]}; \qquad \kappa_2 = \tan^{-1}(\nu \cdot \pi) \cdot \kappa_1. \tag{A.11}$$

The upper bound on the fixed cost, $\bar{F}$, controls the extent of price rigidity. A higher $\bar{F}$ allows for higher menu cost values, subsequently making price adjustments more difficult. The curvature parameters (ω, ν), are chosen so that $G(F)$ is monotonically increasing. As noted above, ψ governs the fraction of flexible-price firms, and thus increasing this value leads to a larger number of small price changes and a higher overall frequency of price adjustment. Detailed specifications of these parameters are provided in Table 1.

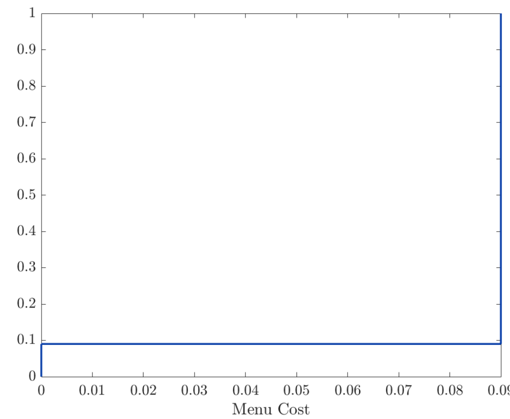
Macro Parameterization In the macro parameterization, the model simplifies the adjustment cost specification compared to the baseline case. As presented in (A.12), under this revised approach, firms are faced with two scenarios: *i)* with probability ψ, they draw zero menu cost, allowing for free price adjustment, *ii)* with probability $1-\psi$, they incur a fixed cost, denoted as $\bar{F}$, when adjusting their prices. The distribution of the new cost parameterization is shown in Figure A.5b. Corresponding values of ψ and $\bar{F}$ are presented in Table 6.

$$G(F) = \begin{cases} 0 & \text{with prob. } \psi \\ \bar{F} & \text{with prob. } 1-\psi \end{cases} \tag{A.12}$$

Figure A.5: Menu Cost Distribution



(a) Baseline



(b) Macro Paramaterization

Note: This figure shows the non-degenerate distribution of price adjustment costs: panel (a) baseline scenario and panel (b) is for macro parameterization.

A.4.4 Computational Algorithm

This section outlines the computation of the SREE (Stochastic Rational Expectations Equilibrium). In this SREE framework, while solving the price-setting problem of the sellers, all state variables are considered exogenous with the sole exception of the aggregate price level.[63]

The algorithm hinges on treating the aggregate price level as an equilibrium object. Particularly, each seller in the economy holds a belief about the overall price. At equilibrium, these beliefs coincide with the actual aggregate price. Therefore, the solution strategy focuses on determining the equilibrium aggregate price function, denoted as $P(M, S)$. As shown in (A.9), this function is derived by aggregating the firm level prices which are obtained by solving the seller's problem. For the ease of exposition, we use the (S, s) notation as described in Appendix A.2, where $S = (x, \mu_Q, disp_Q, disp_D)$ represents the aggregate state space, and $s = (z, \alpha, F)$ indicates the idiosyncratic state space.

The computational process involves directly solving a nonlinear system of equations, which are the first-order conditions of the firm's *ex ante* and *ex post* problems. In what follows, we provide a concise overview of the steps involved in the solution algorithm.

Step 1 – Initialize Aggregate Price Function Begin with an initial guess of the aggregate price function, denoted as $P^{(0)}(M, S)$. This function is represented by a four-dimensional matrix, which corresponds to the four elements of the aggregate state variable set $S = (x, disp_Q, disp_D, \mu_Q)$.

[63]See Appendix A.3 for the full definition of SREE in this linear quadratic setting.

Step 2 – Calculate the Updated Aggregate Price Function Compute the new implied aggregate price function, $P^{(1)}(M, S)$, by executing the following procedures:

i. **Nonlinear System Solution:** Solve the nonlinear system governed by (A.2) – (A.6). Note that (A.3) is not a standalone equation *per se*, but rather a set of equations each pertaining to a specific point in the state space. Solving the system yields *ex ante* price, $\bar{p}(M)$ and a set of *ex post* prices, $\tilde{p}(M, S, s)$.

ii. **Calculation of Adjustment and Non-adjustment Values:** For each point in the state space, use the *ex ante* price, $\bar{p}(M)$ and the *ex post* price set $\tilde{p}(M, S, s)$ to determine the values of adjustment $W^a(M, S, s)$ and non-adjustment $W^n(M, S, s)$, as defined in (A.4) and (A.5), respectively.

iii. **The Adjustment Decision:** Compare the adjustment value $W^a(M, S, s)$ with the non-adjustment value $W^n(M, S, s)$ at each point in the state space. Record the maximum value for each scenario and note whether it results from adjustment or non-adjustment.

iv. **Forming the Realized Price Matrix:** Based on the decision regarding price adjustment, select the corresponding price (*i.e. ex ante* price, $\bar{p}(M)$ and the *ex post* price set $\tilde{p}(M, S, s)$) and construct the realized price matrix for each point in the state space (M, S, s).

v. **Derivation of the Updated Aggregate Price Matrix**: Employ the probability of each unique state to calculate the weighted average of individual prices to obtain the new aggregate price matrix, denoted as $P^{(1)}(M, S)$.

Step 3 – Assess Convergence Evaluate the convergence of the aggregate price function by measuring the distance between $P^{(0)}(M, S)$ and $P^{(1)}(M, S)$. If this distance falls within the predetermined error tolerance band, the aggregate price function is considered to have converged, yielding the price policy functions. If convergence is not achieved, update the initial guess of the aggregate price function by setting $P^{(0)}(M, S) = P^{(1)}(M, S)$ and repeat the process from Step 1. Continue iterating until convergence is attained.

Note that there is no approximation involved in the solution algorithm. The approach directly solves a system of equations to find a SREE. So unlike an approach based upon Krusell and Smith (1998), there are no moments *per se* used to characterize an equilibrium.