TFPR: Dispersion and Cyclicality*

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Abstract

This paper studies the determinants of TFPR, a revenue based measure of total factor productivity. Recent business cycle models are built upon the assumption of countercyclical dispersion in TFPR. But, the distribution of TFPR is endogenous, dependent upon exogenous shocks and the endogenous determination of prices. An overlapping generations model with monopolistic competition and state dependent pricing is constructed to study the factors that shape the TFPR distribution. The empirical focus is on three key data patterns: (i) countercyclical dispersion of TFPR , (ii) countercyclical dispersion of price changes and (iii) countercyclical frequency of price adjustment. The analysis uncovers two interesting scenarios in which these moments are matched. One arises in the presence of shocks to the dispersion of TFPQ, a quantity based measure of total factor productivity, along with a negatively correlated change in the mean of TFPQ. The second arises if the monetary authority responds to shocks to the dispersion of TFPQ by "leaning against the wind". Due to state contingent pricing, the model is nonlinear. Simple correlations mask these nonlinearities of the underlying economy. The real effects of monetary innovations are state dependent, with monetary policy less effective in recessions.

1 Motivation

There is ample evidence that the cross-sectional dispersion of productivity is countercyclical.[1] Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) use this feature of the data as a key input into a model of aggregate fluctuations, highlighting the effects of uncertainty.[2] Relatedly, Vavra (2014) provides evidence that in recessions price changes are more dispersed and the frequency of price adjustment is higher. He argues that these patterns can be reproduced in a model with variations in the volatility of firm level productivity as these fluctuations induce some sellers to adjust prices upwards and others to adjust downwards.[3]

While the evidence of countercyclical dispersion of productivity is relatively incontrovertible, debate continues on the source of this pattern. One point of concern, raised for example in Berger, Dew-Becker,

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[1]See the evidence and discussion in, for example, Kehrig (2011), Bachmann and Bayer (2014), and Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). The evidence is presented as changes in the distribution of total factor productivity and/or the correlation in the dispersion of total factor productivity with a measure of economic activity. Bachmann and Bayer (2014) provide complementary evidence from German data.

[2]Here there is an important distinction between uncertainty and dispersion. Uncertainty refers to an *ex ante* situation of not knowing, say, some moment of the distribution of a random variable, such as not knowing the future variance. Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) contains both uncertainty and dispersion effects.

[3]His calibration relies upon the same measures of dispersion as Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). The connection between firm specific shocks and the distribution of price changes is highlighted in Golosov and Lucas (2007) as well.

and Giglio (2020), is the limited evidence that uncertainty drives these patterns. Further, as argued in this paper, there is an important distinction between measurement and theory that is ignored. This leads to the central question of the paper: what drives the cyclicality of the dispersion in TFPR?

Note that the question pertains to TFPR, a revenue based measure of productivity, and not the quantity-based measure of productivity, TFPQ. The facts presented in Foster, Haltiwanger, and Syverson (2008) make clear that: (i) the distributions of TFPQ and TFPR differ and (ii) the distribution of TFPR is not degenerate. The first point implies that any model attempting to study both of these distributions needs to rationalize the difference between TFPR and TFPQ. Further, that model, following the discussion in Hsieh and Klenow (2009), must explain why the distribution of TFPR is not degenerate.

The distinction between TFPR and TFPQ is important for understanding the existing literature and our contribution. The empirical findings regarding the cyclicality of productivity rely on measurements of TFPR not TFPQ. Yet, the models routinely focus on variations in the dispersion of TFPQ as a driving force and equate these with variations in the dispersion of TFPR. The distinction between the dispersion in TFPQ and TFPR is not a component of the analysis in prominent contributions, such as Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) or Vavra (2014). Our analysis instead uncovers conditions for countercyclical variation in TFPR dispersion but does not support the view that it necessarily arises from countercyclical variations in the dispersion of TFPQ.

To be clear, this paper does not contest the cyclicality of TFPR dispersion. Rather, it studies the determinants of this cyclicality along two general possibilities. The first, quite naturally, is that variations in the dispersion of TFPQ underlie variations in the dispersion of TFPR. The second focuses on the effects of price determination directly on the dispersion of TFPR. That is, changes in the distribution of prices can generate movements in the dispersion of TFPR, holding the distribution of TFPQ fixed.

Our model and quantitative analysis explicitly incorporate the distinction between TFPQ and TFPR, building on Foster, Haltiwanger, and Syverson (2008). As prices form the bridge between the distributions of TFPR and TFPQ, price setting plays a central role in our analysis. From Table 1 of Vavra (2014), the standard deviation of prices is countercyclical. This suggests the possibility that price movements, in response to shocks, contribute to the cyclicality of the dispersion in TFPR. Moreover, price stickiness directly creates a non-degenerate distribution of TFPR, so that other types of frictions or wedges are not needed.[4] Thus, ignoring the distinction between TFPQ and TFPR is not only at variance with the evidence but also misses the contribution of endogenous price setting.

Our central question is addressed through a model of state dependent pricing, with heterogenous firms, to obtain a mapping from the distribution of TFPQ to the distribution of TFPR. In contrast to the flexible price case, state dependent pricing due to menu costs introduces both extensive and intensive margins of pricing decisions and thus allows for a variety of factors, both monetary and real, to influence the distribution of TFPR.

The framework for analysis is an overlapping generations model with monopolistic competition and sticky prices, specified in section 2. Young agents have market power, set prices *ex ante* and can, at a cost, change them *ex post*, once the various shocks (productivity, tastes and monetary) along with the menu cost are realized. Old agents take money earnings from youth as well as monetary policy induced transfers and spend them on a variety of goods. The analysis is conducted through a stationary rational expectations equilibrium for this environment. The model allows aggregate shocks to the dispersion of idiosyncratic demand, the mean and dispersion of productivity and to the money supply.

[4]We are grateful to John Haltiwanger for emphasizing this point to us.

Admittedly this is not a common framework for studying price determination in an aggregate model, but it has a number of distinct advantages. First, individual choice problems, in particular the state dependent pricing problem of sellers is very tractable. Second, we are able to obtain a full characterization of a stationary rational expectations equilibrium, including a wide variety of shocks to technology, tastes and the money supply. Finally, there are no approximations in the quantitative analysis. Instead the determination of the effects of the various shocks is through the stationary rational expectations equilibrium.

An apparent weakness of the approach is that the *ex post* pricing decisions of sellers have no dynamic component. However, we find that in a parameterized version of the model, the price setting behavior of sellers in this model mimics key features of the price setting from models with infinitely lived sellers. In this sense, while the model misses some dynamic elements of forward looking pricing, it does capture the essence of the state dependent pricing.

The overlapping generations model provides a framework for conducting quantitative experiments in a stochastic equilibrium setting. Section 3 presents the quantitative model. The calibration is based on a steady state of the model. The pricing moments for the calibration come from Vavra (2014) while Foster, Haltiwanger, and Syverson (2008) is used for moments on the dispersion of TFPQ and TFPR. The calibration pins down the elasticity of demand as well as the relative importance of shocks to the dispersion of demand and the dispersion of TFPQ. We find that a much higher dispersion in technology shocks relative to demand is needed to match the moments. The distribution of menu costs comes from Dotsey and Wolman (2019).

The calibrated model has a particularly important feature, common to models of state dependent pricing: a U-shaped hazard. In the parlance of Caballero and Engel (1993), the adjustment rates are very high when the absolute value of the gap between the actual and desired price is large and very low when this gap is near zero. That characterization applies almost directly to our model since price setting is a static problem. This nonlinear hazard creates nonlinear responses in the economy, particularly in the presence of monetary shocks, and this nonlinearity pervades the quantitative analysis.

The model is assessed by its ability to mimic key data features: *(i)* countercyclical dispersion in TFPR, *(ii)* countercyclical frequency of price adjustment and *(iii)* countercyclical dispersion in price changes. Our first set of findings is negative. Taken individually, none of the sources of aggregate variation we consider can reproduce the data patterns of countercyclical dispersion in TFPR, the frequency of price adjustment and the dispersion of price changes.

A natural starting point, in keeping with the literature, is to consider exogenous variations in the dispersion of TFPQ as the source of countercyclical dispersion in TFPR. Consistent with Vavra (2014), shocks to the dispersion of TFPQ do succeed in matching data patterns *(ii)* and *(iii)*. But, in our model, shocks to the dispersion of TFPQ are procyclical and produce procyclical variation in the dispersion of TFPR. **Thus, shocks to the dispersion of TFPQ alone are unable to match data patterns.**

This contrasts with Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). In their setting, uncertainty plays a prominent role. However, recent findings of Berger, Dew-Becker, and Giglio (2020) and Dew-Becker and Giglio (2020) casts some doubt on the central role of variations in uncertainty as the source of the countercyclical dispersion.[5] This finding also contrasts with Vavra (2014) who also asserts the countercyclical dispersion of TFPQ, without distinguishing it from the distribution of TFPR.

Due to price setting behavior, TFPR dispersion responds to other shocks, including variations in the money supply, the distribution of idiosyncratic demand and the mean of productivity. Analyzed independently, these shocks generate cyclical movements in the dispersion of TFPR, both through effects on the

[5] The effects of uncertainty in our environment are studied in Subsection 6.4.

dispersion of prices and, perhaps more interestingly, through the covariance of prices and the firm specific productivity shock. But the model moments produced from these sources of fluctuations do not match data patterns.

The second set of findings relate to experiments that combine these sources of variations. **A combination of a shock (increase) to the dispersion of TFPQ combined with a (decrease) in the mean of TFPQ, matches the data patterns.** Essentially the dispersion of TFPR is driven by the dispersion of TFPQ while output movements depend more on the mean of TFPQ. The combination of shocks provides a mechanism that drives a wedge between the dispersion of TFPR and that of TFPQ. Combining these two shocks is crucial: an increase in the dispersion of TFPQ alone cannot capture the empirical pattern of countercyclical dispersion in TFPR.

These results are supportive of findings in the literature. In order to avoid negative correlation between consumption and investment in the face of an uncertainty shock, Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), combine a shock to the dispersion of TFPQ with a reduction in average TFPQ. Vavra (2014) employs a variation of this specification. But, again, the mechanisms are different. We do not rely on uncertainty shocks. Further, our shocks relate to the distribution of TFPQ not the endogenous distribution of TFPR so that the pricing decisions impact the TFPR distribution.

In addition to this combination of shocks, we allow the monetary authority to respond to exogenous variations in the mean and dispersion of TFPQ as well as to changes in the dispersion of demand, thus creating a comovement with the money shocks. Through its response to these shocks, the monetary authority induces a correlation between output and money innovations. If the monetary authority "leans against the wind", i.e. tightens monetary policy when output is high, in the face of either shocks to the dispersion or mean of TFPQ, then the data patterns of countercyclical dispersion in TFPR, the dispersion of price changes and the frequency of price adjustment emerge.

Section 6 looks at additional properties of the model economy. First, we highlight the nonlinear properties of the equilibrium response to shocks, which comes from the U-shaped price adjustment hazard. For this, we compute conditional correlations to indicate how variables comove as a function of the aggregate state (expansion or contraction). Second, we ask whether the model generates state dependence in the effects of monetary shocks, as documented in Tenreyro and Thwaites (2016). In fact, we do find evidence that money shocks have a bigger impact during expansions. But, in contrast to Tenreyro and Thwaites (2016), this holds regardless of whether expansions (contractions) are defined as positive (negative) growth in output or as above (below) average in the level of output. Finally, we introduce uncertainty into the framework, distinguished from *ex post* changes in dispersion. We find no effects of uncertainty.

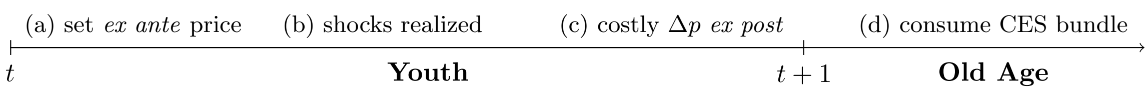
2 Equilibrium Model of State Dependent Pricing

We study the determination of the distribution of TFPR in an infinite horizon overlapping generations model with differentiated products and market power. Agents live for two periods, youth and old age. Generation t young agents produce and, when old, these agents consume a basket of goods produced by the next generation of young producers. Saving occurs through the holding of fiat money. The quantity of fiat money is stochastic, representing monetary shocks.[6]

[6] This presentation focuses on a version of the model in which there are aggregate shocks to the model supply as well as idiosyncratic shocks to seller productivity and to the cost of price adjustment. The quantitative analysis adds other sources of variation. The extension of the model to include these additional shocks appears in Subsection 2.5 and Appendices 8.2 - 8.3.

As noted earlier, this is admittedly not the standard framework for the analysis of state dependent pricing. The benefit of the model is the simplicity of the choice problems coupled with an equilibrium analysis that allows experiments without the introduction of unexpected shocks to money and/or the distributions of productivity and demand. This point is central since understanding how the economy responds to shocks is the key question of the paper. Answering this in full articulated stochastic economy is therefore necessary.[7]

Figure 1: Time Line: Generation t



The sequence of choices is shown in Figure 1. Generation t young agents set a price *ex ante*, prior to the determination of any shocks but dependent on the history of the economy, summarized in equilibrium by the stock of money inherited from the previous period. This indicated by (a) on the timeline. At point (b) shocks to the aggregate money supply as well as to idiosyncratic productivity and idiosyncratic menu costs are realized. Given these realizations, sellers have an option of *ex post* price adjustment, indicated by point (c). This is the step that generates heterogeneous price setting, both on the extensive margin (to adjust the *ex ante* price or not) and in the event of adjustment, the intensive margin choice of what price to set.

There are a couple of features of the model economy worth highlighting. First, the price setting stage is interdependent in that the optimal price of one seller depends on the *ex post* price of the adjusters as well as the *ex ante* price of the non-adjusters. As usual, sellers meet demand at their posted price.

Second, the *ex post* decision on price adjustment depends on the realization of all shocks. In this way, the dispersion of the distribution of productivity shocks impacts the frequency of adjustment and thus the real effects of money shocks.

Third, the inclusion of two forms of idiosyncratic shocks, one to productivity and the other to the adjustment costs, creates an interesting tension in the adjustment decision. A seller with a very large productivity shock might be induced to adjust the *ex ante* price but may draw a high adjustment cost and thus not reset its price. This tension has implications for the equilibrium effects of money shocks as the selection into price adjustment depends on all of these shocks. Further, for our purposes, the relationship between the exogenous TFPQ distribution and the endogenous TFPR distribution depends on the price setting behavior of sellers.

Fourth, as in Lucas (1972), in the absence of price stickiness, there would be a stationary rational expectations equilibrium in which money was neutral. This is because money transfers are made to the old in proportion to money holding earned in youth. And, as in that paper, the analysis rests on the coexistence of real and nominal shocks. But, in our setting the friction of costly price adjustment replaces his assumption of imperfect information.[8]

[7] Many other models do not have an equilibrium demand for money and/or do not study the effects of money and other aggregate shocks in an equilibrium framework.

[8] Of course, in his model the real shock was to the fraction of sellers in a particular market while we focus on productivity shocks.

2.1 Choice of Old Agents

Lifetime utility is represented by $u(c) - g(n) = \frac{c^{1-\sigma}}{1-\sigma} - g(n)$. Here c is a CES aggregator given by $c = \left(\sum_i c^{i\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$, with $\varepsilon > 1$.[9] The function $g(\cdot)$ is increasing and convex in hours worked, with $0 \leq n \leq 1$. As we shall see, both the substitutability between products as well as the curvature in the disutility of work play important roles in the pricing decisions of young agents, particularly the choice of *ex post* adjustment.

When old, agents take their money holdings from income earned in youth and allocate it across goods to maximize $u\left(\left[\sum_i (c^i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right)$, subject to a budget constraint of $\sum_i c^i p^i = M$ where M is their nominal income and p^i is the money price of good i.[10]

For these preferences, the demand for good i is given by

$$c^i = d(p^i, P, M) = \left(\frac{p^i}{P}\right)^{-\varepsilon} \frac{M}{P}. \tag{1}$$

Here P is an aggregate price index defined as $P = \left(\sum_i (p^i)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$. Note that the only shock to demand is from variations in the stock of money, M.

Let $V(\frac{M}{P})$ be the value of the solution to the optimization problem of an old agent with nominal income of M with prices given by P. Given the definition of c,

$$V(\frac{M}{P}) = u\left(\left[\sum_i \left((\frac{p^i}{P})^{-\varepsilon}\frac{M}{P}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right) = u\left(\left[\sum_i \left((\frac{p^i}{P})^{-\varepsilon}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \frac{M}{P}\right) \tag{2}$$

with P given above. From this, the marginal value of nominal income is given by $V_M = \frac{u'(c)}{P}$.

At this point, these are generic demands and values for an old age given nominal income and prices. These values summarize the outcome of the choice problem for old agents in period t at point (d) of the time line in Figure 1. We will take this structure and use it to study the choices of young agents in the OG framework, summarizing the utility they obtain when old through $V(\frac{M}{P})$.

2.2 Choice of Young Agents

We start with the pricing decisions of generation t young agents. When young agents choose the price of their product *ex ante*, they take into account the option, at a fixed cost, of adjusting their price *ex post*. These are points (a) and (b) in Figure 1. Since this is a model of a menu rather than a quadratic cost at the micro-level, the *ex ante* price will influence the frequency of adjustment but not the *ex post* price conditional on adjustment.

As is common in the literature, see for example Galí (2015), agents are assumed to meet the demand forthcoming at their price. Thus the prices they set will determine their nominal income in youth.

This nominal income is held over time in the form of money to purchase consumption goods when old. Holdings of money are altered through monetary policy. Thus in our framework, money holdings and monetary policy interventions are made explicit.

[9] We normalize the number of young agents and thus products to 1. With the CES assumption, markups are constant. This puts aside another potentially interesting interaction between the level of economic activity and prices.

[10] To simplify the notation, the time subscript is repressed. The money holdings come from income earned in youth as money is the store of value in this economy. Many other general equilibrium models, such as Dotsey, King, and Wolman (1999), impose money demand. In Golosov and Lucas (2007), money is in the utility function.

To study the pricing choice, consider the *ex post* decision of generation t sellers.[11] If they choose to adjust, these sellers choose a price $\tilde{p}$ to solve

$$W^a(z_t, M_{t-1}, x_t, P_t) = max_{\tilde{p}} E_{x_{t+1},P_{t+1}} V((R(\tilde{p}, P_t, M_t))x_{t+1}/P_{t+1}) - g(\frac{d(\tilde{p}, P_t, M_t)}{z_t}). \quad (3)$$

Here the demand, denoted $d(\tilde{p}, P_t, M_t)$ and specified in (1), is the spending of the old agents on the product of this seller. The function $V((R(\tilde{p}, P_t, M_t))x_{t+1}/P_{t+1})$ is given by (2) with, in that notation, $M = R(\tilde{p}, P_t, M_t)x_{t+1}$ being the nominal revenue earned as a seller in period t supplemented by the period $t+1$ money shock and $P = P_{t+1}$, the period $t+1$ aggregate price level.

Since this decision is made *ex post*, the value and the price depend on the current state: (z_t, M_{t-1}, x_t, P_t). Here z_t is the current idiosyncratic productivity shock, M_{t-1} is the inherited money supply, x_t is the money shock and P_t is the aggregate price level, determined in equilibrium as described below.

There is also a seller specific menu cost, denoted F, that influences whether adjustment occurs or not but not the price selected given adjustment. The adjustment cost is written as a utility loss. This specification has a convenient property that the optimal price is independent of the adjustment cost. So, the extensive margin of adjustment will depend on the realized menu cost and idiosyncratic productivity but the intensive margin does not so that the price dispersion of adjusters reflects only heterogeneity in z_t.

In this formulation, the menu cost F has a cumulative distribution function denoted $\Omega(\cdot)$. The inclusion of stochastic menu costs weakens the selection effect, i.e. the dependence of the pricing decision on z. As we shall see with the calibrated model, this implies that the probability of price adjustment is an increasing function of the absolute value of the idiosyncratic technology shock but it is not a step-function. That is, there are no bounds on z such that price adjustment occurs iff z is outside those bounds.

Notice that the price set by these sellers is independent of any price they may have set *ex ante* so that the *ex ante* choice does not appear in the state space. Importantly, once the cost of adjustment is incurred, the price reflects both the monetary shock and seller specific productivity. In this sense, there is an underlying complementarity at work. If a seller pays an adjustment cost to respond to one type of shock, then the marginal cost of responding to another type of shock is zero. This is important for the analysis that follows as it explains why price dispersion and thus TFPR dispersion is influenced by monetary policy.

With the production function of $y = zn$, the labor input of the seller is given by $\frac{d(\tilde{p},P_t,M_t)}{z_t}$. As the seller meets all demand, the labor input varies inversely with productivity, **given** demand.

The first-order condition is

$$E_{x_{t+1},P_{t+1}}\left(u'(c_{t+1})x_{t+1}\frac{d(p_t, P_t, M_t)(1-\varepsilon)}{P_{t+1}}\right) = g'(\frac{d(p_t, P_t, M_t)}{z_t})\left(-\varepsilon\frac{d(p_t, P_t, M_t)}{p_t z_t}\right). \quad (4)$$

Denote this *ex post* optimal price by $p_t = \tilde{p}(z_t, M_{t-1}, x_t, P_t)$ for all a seller with realized productivity z_t.

This is the standard condition for optimal price setting, equating marginal revenue with marginal cost.[12] But in this overlapping generations model, marginal revenue is determined by the marginal utility of the future consumption that can be acquired with the additional money income. And that income is itself impacted by future monetary policy, through the stochastic transfer x_{t+1}.

[11] That is, we solve the agents problem at point (b) and use this solution to study the *ex ante* problem at point (a).

[12] To understand this condition in a static setting, let $d = (\frac{p}{P})^{-\varepsilon}y$ be the level of produce demand if the seller sets the price p and the aggregate price is P and the level of real spending is y. So $d_p = -\varepsilon\frac{d}{p}$. Further, revenue is given by $R = pd = p^{1-\varepsilon}(\frac{1}{P})^{-\varepsilon}y$. Hence $R_p = (1-\varepsilon)d$. The left side of (4) is the product of R_p and $\frac{u'(c_{t+1})x_{t+1}}{P_{t+1}}$. The right side is the product of d_p and the marginal disutility of work, $g'(\frac{d(p_t,P_t,M_t)}{z_t})\frac{1}{z_t}$.

Alternatively, if the seller does not adjust, then expected lifetime utility is given by:

$$W^n(z_t, M_{t-1}, x_t, P_t, \bar{p}) = E_{x_{t+1},P_{t+1}} V((R(\bar{p}, P_t, M_t))x_{t+1}/P_{t+1}) - g(\frac{d(\bar{p}, P_t, M_t)}{z_t}). \quad (5)$$

Here, expected utility depends on the preset price, $\bar{p}$.

Given this, consider the *ex ante* choice. When this price is set, the young agent just knows the money supply from the past. Let $W^{xa}(M_{t-1})$ be the value to a young agent of setting the price *ex ante*. The value is given by:

$$\begin{aligned} W^{xa}(M_{t-1}) &= max_{\bar{p}} E_{(z_t, x_t, x_{t+1} P_t, P_{t+1})}[(1 - \Omega(F^*(\Omega_t)))W^n(z_t, M_{t-1}, x_t, P_t, \bar{p}) + \\ & \int_0^{F^*(\Omega_t)} W^a(M_{t-1}, x_t, P_t) - F]d\Omega(F) \end{aligned} \quad (6)$$

where $F^*(z_t, M_{t-1}, x_t, P_t)$ is the critical menu cost in state (z_t, M_{t-1}, x_t, P_t) such that price adjustment occurs iff $F \leq F^*(z_t, M_{t-1}, x_t, P_t)$. Let $\bar{p}(M_{t-1})$ denote the optimal *ex ante* choice.

2.3 SREE

The analysis is based on a stationary rational expectations equilibrium (SREE) with valued fiat money.[13] The current aggregate state is represented as (M, x) where M is the inherited money supply and x is the current shock, so that the current money supply is Mx. At the individual supplier level, productivity and the cost of price adjustment are the two elements in the idiosyncratic state: (z, F). At this point of the analysis, the distribution of the idiosyncratic shocks is fixed and thus not in the state vector. An equilibrium is defined and characterized given that distribution.

There are four state dependent functions to be determined. The *ex ante* price set knowing only M is denoted $\bar{p}(M)$. The *ex post* price set by sellers who choose to adjust their price is given by $\tilde{p}(M, z, x)$, indicating the price depends on both the realized money shock and productivity. There is a critical level of the adjustment cost, $F^*(M, x, z)$, such that adjustment occurs iff $F \leq F^*(M, x, z)$. Finally, the *ex post* money price of goods, $P(M, x)$, clears the goods market.

Definition 1 (SREE) A SREE is a set of functions $(\bar{p}(M), \tilde{p}(M, z, x), F^*(M, x, z), P(M, x), W^n(M, x, z), W^a(M, x, z))$ such that:

- $\bar{p}(M)$ solves the *ex ante* pricing problem given the state dependent price index $P(M, x)$;

  $$\bar{p}(M) = argmax_p E_{x,z,x'} V((R(p, P(M, x), Mx)x')/P(Mx, x')) - g(\frac{d(p, P(M, x), Mx)}{z}). \quad (7)$$

  for all M.

- $\tilde{p}(M, x, z)$ solves the *ex post* pricing problem:

  $$\tilde{p}(M, x, z) = argmax_p E_{x'} V\left((R(p, P(M, x), Mx))x'/P(Mx, x')\right) - g(\frac{d(p, P(M, x), Mx)}{z}) \quad (8)$$

  given the state dependent price vector, $P(M, x)$, for all (M, x, z).

[13] The more general SREE -including shocks to the distribution of idiosyncratic productivity, as well as other aggregate shocks- is presented in Appendix 8.2.

- At the critical adjustment cost, $F^*(M,x,z)$, the seller is just indifferent between adjusting and not:

$$F^*(M,x,z) \equiv W^n(M,x,z) - W^a(M,x,z) \tag{9}$$

for all (M,x,z), with $W^a(M,x,z)$ given by:

$$W^a(M,x,z) = E_{x'}V\left((R(\tilde{p}(M,x,z),P(M,x),Mx))x'/P(Mx,x')\right) - g\left(\frac{d(\tilde{p}(M,x,z),P(M,x),Mx)}{z}\right) \tag{10}$$

and $W^n(M,x,z)$ given by

$$W^n(M,x,z) = E_{x'}V((R(\bar{p}(M),P(M,x),Mx))x'/P(Mx,x')) - g\left(\frac{d(\bar{p}(M),P(M,x),Mx)}{z}\right). \tag{11}$$

- $P(M,x)$ is the aggregate price index in state (M,x) given by:

$$P(M,x) = [E_z(1-\Omega(F^*(M,x,z)))\bar{p}(M)^{1-\varepsilon} + E_z(\Omega(F^*(M,x,z))\tilde{p}(M,x,z)^{1-\varepsilon})]^{\frac{1}{1-\varepsilon}} \tag{12}$$

where $d(\bar{p}(M),P(M,x),Mx) = \left(\frac{\bar{p}(M)}{P(M,x)}\right)^{-\varepsilon} Y$ and $d(\tilde{p}(M,x,z),P(M,x),Mx) = \left(\frac{\tilde{p}(M,x,z)}{P(M,x)}\right)^{-\varepsilon} Y$. Here $Y = \frac{Mx}{P(M,x)}$ is the equilibrium determined real value of money holdings.

2.4 Equilibrium Properties

This section briefly describes properties of a SREE, both at the aggregate and individual seller level. These properties are made more explicit in the quantitative analysis.

2.4.1 Money Non-Neutrality

There are two main properties of a SREE that are verified in the analysis that follows.

Proposition 1 *There exists a SREE in which: (i) real quantities are independent of M since all prices set ex ante and ex post are proportional to M and (ii) real quantities are not independent of x.*

First, the inherited money supply is neutral: i.e. prices are proportional to M and all real quantities are independent of M. Formally, this amounts to guessing and verifying that there is a SREE in which $\bar{p}(M) = QM$ where Q is an unknown constant and $\tilde{p}(M,x,z) = M\tilde{\phi}(x,z)$. From this all relative prices and thus quantities demanded (and thus supplied) are independent of M.

The second property is money non-neutrality. If prices were not costly to adjust, i.e. the distribution of F was degenerate at $F = 0$, then there would exist a SREE with prices proportional to Mx. In this case, real quantities would be independent of the current money supply, Mx. But, in the presence of non-degenerate menu costs, as long as some sellers choose not to adjustment their prices *ex post*, a SREE with prices proportional to Mx cannot exist simply because the preset price, $\bar{p}$, must be independent of x.[14]

[14] Formally, this requires that the support of menu costs be large enough so that even if all other sellers adjust their prices *ex post*, the remaining seller, for any x, will have a high enough adjustment cost so that adjustment will not occur. See Ball and Romer (1991) for a discussion of this related to multiplicity of equilibria.

2.4.2 Productivity Measures

Returning to the theme of productivity measures, the difference between TFPQ and TFPR is straightforward to characterize. Here, z corresponds to the TFPQ measure of productivity. It is exogenous to the seller. The variable $\frac{zp}{P}$ is TFPR, where $p \in \{\tilde{p}(M, x, z), \bar{p}\}$ reflects the seller's pricing choice and P is the aggregate price.[15] Though the distribution of TFPQ is exogenous, the distribution of TFPR is endogenous as prices are set by sellers. Thus the distribution of TFPR responds to shocks insofar as sellers adjust prices in response to those shocks.

The price stickiness as well as the limited reallocation of labor across production sites help to shape the distribution of TFPR. To illustrate, consider a static, flexible price version of the model where $TFPR = pz = q^{-\eta}z = z^{1-\eta}n^{-\alpha\eta}$ where the production function is $q = n^{\alpha}$. From the first order condition with respect to n, if marginal cost of labor is ω, we have

$$(1-\eta)\alpha n^{(-\alpha\eta+\alpha-1)} z^{1-\eta} = \omega.$$

At $\alpha = 1$, this condition becomes $(1-\eta)n^{-\eta}z^{(1-\eta)} = \omega$ which holds for all z. This implies that TFPR is given by $\frac{\omega}{1-\eta}$ and hence is independent of z. So, in this limiting case, variations in the distribution of TFPQ would not impact the distribution of TFPR. In our model, both price stickiness and non-linear production costs along with labor immobility will contribute to the non-degenerate distribution of TFPR.

2.4.3 Seller Choices

In equilibrium, aggregate real output is given by: $Y(x) = \frac{Mx}{P(M,x)} = \frac{x}{\varphi(x)}$, where, using the first part of Proposition 1, $P(M, x) = M\varphi(x)$. Thus the response of output to money shocks will depend on $\varphi(x)$, in the absence of other aggregate shocks. This function summarizes the responses by sellers to monetary shocks. It captures both the extensive margin of adjustment, i.e. the fraction of sellers resetting their price *ex post*, as well as the intensive margin of the optimal price to set.

Note that from the first property of the equilibrium, the inherited money supply, M, is completely neutral. It has no effect on either the extensive or intensive margins.

The money shock impacts both margins. In terms of adjustment frequency, more extreme shocks generate a higher fraction of sellers choosing to adjust. Further, for those sellers adjusting, the *ex post* will depend on the money shock. But, importantly, it will not be proportional to x. Thus the non-neutrality arises on both the extensive and intensive margins.

There is an important feature of our model that ties directly with the line of research which studies the frequency of price adjustment as a function of a gap between actual and desired prices. Caballero and Engel (2007) discusses this approach and cites numerous related papers. Our model, with its one time price adjustment, fits exactly into that framework. This can be seem from (9), where the difference in the values between adjusting and no adjusting are used to determine the critical adjustment cost. These difference in values is directly related to the gap between the *ex ante* price, $\bar{p}(M)$, and the state contingent *ex post* price, $\tilde{p}(M, x, z)$.[16]

[15] Since TFPQ is measured directly in simulated data, there is no need to infer TFPR from revenue and thus no discussion of output or revenue factor shares. See the discussion of these measurement issues in Decker, Haltiwanger, Jarmin, and Miranda (2019).

[16] This is explored in the quantitative analysis of the linear quadratic economy.

2.5 Additional Aggregate Shocks

Thus far the analysis includes only a single aggregate shock. This was simply to enhance the transparency of the presentation. Introducing additional sources of aggregate uncertainty into this framework is direct.

Appendix 8.2 presents the more general economy in which there is an aggregate state, S, that includes shocks to the money supply, variations in the distribution of z and relative demand shocks.[17] The optimization problems of agents as well as the definition of equilibrium is directly extended to this enhanced environment. It is the basis of the quantitative analysis that follows.

Two shocks to the distribution of TFPQ are studied. One is the traditional TFPQ shock in which the mean of the z distribution, denoted μ_Q, is stochastic. In this case, the output of a seller becomes $y = \mu_Q zn$. The second, which follows the motivation of the paper is a shock to the dispersion of z, denoted $disp_Q$, holding the mean fixed.

Finally, the model is extended to incorporate idiosyncratic demand shocks.[18] This provides a direct shock to the dispersion of TFPR, through demand, and independent of the dispersion in TFPQ. These are modelled as seller specific shifts in demand. As discussed below, these shocks differ from the idiosyncratic productivity shocks, particularly when prices are sticky. In terms of aggregate shocks, we study mean preserving spreads in the distribution of demand shocks, denoted $disp_D$. Variations in the mean level of nominal spending are studied through the money shocks.

Through all of these extensions of the stochastic framework, the basic structure of the model and the insistence on a SREE is maintained. Further, the numerical solution operates directly on the conditions for a SREE, without the need for linear approximations.

3 Quantitative Analysis

The estimation of underlying parameters is best left for an empirical exercise that studies price setting by infinitely lived firms matching high frequency observations on price and quantities. At this point, such an ideal data set is not available. Our goal is more modest and should be considered as an extended quantitative example allowing us to focus on the determination of the distribution of TFPR in an equilibrium model.

That said, the quantitative version of the OG pricing model has features of the standard macroeconomic pricing models, including both the Calvo model and state dependent pricing problems. In the Calvo model, as in the OG structure, the probability of price adjustment and the price set conditional on adjustment, are independent of the previously set price. Further, in some specifications, such as Christiano, Eichenbaum, and Evans (2005), price setters who do not adjust get to freely reset prices based upon past inflation. This added feature further reduces the role of history for price setting. In the OG model, this is captured by period t price setters choosing a price that is proportional to the inherited money supply.

Further, as discussed in Klenow and Malin (2010), existing evidence suggests that for individual sellers the likelihood of price adjustment at a particular point in time is independent of the time since last adjustment. Though allowing full state dependence (conditional on paying an adjustment cost), our model also has this history dependent feature as the choices of sellers in period t does not depend on prices in the past.

Price setting in this model also reproduces familiar patterns of state dependent price adjustment. That is the model generates pricing rules for sellers that retain the essential features of the more standard infinitely

[17] The inclusion of the relative demand shocks is motivated by the findings of the importance of this source of variation in Hottman, Redding, and Weinstein (2016).

[18] The augmented model is discussed in Appendix 8.1. See Eslava and Haltiwanger (2020) for discussion of a similar specification.

Table 1: Parameterization

Parameter	Value	Description
Menu Cost Distribution		
ψ	0.053	Probability of zero menu cost
$\bar{F}$	0.033	Upper bound on menu cost
ω	41.9	Curvature parameter
ν	2.8	Curvature parameter
Utility Parameters		
ϵ	2.37	Elasticity of substitution between products
ϕ	2	Elasticity of labor supply
Idiosyncratic Productivity Shock		
σ_z	0.0378	Standard Deviation
Idiosyncratic Taste (Demand) Shock		
σ_d	0.0069	Standard Deviation

lived agent specifications. This is made clear in the discussion of the pricing behavior of sellers below.

The calibration of the model serves two purposes. First, it sets the basis for the quantitative assessment of the cyclical properties of the distribution of TFPR. Second, as the model includes both demand and technology shocks, the analysis contributes to the ongoing discussion of the relative importance of these sources of variation.

3.1 Calibration

The quantitative analysis rests upon a linear-quadratic economy: $u(c) = c, g(n) = \frac{n^\phi}{\phi}$, where ϕ is the elasticity of labor supply.[19] For the baseline, $\phi = 2$. Varying this elasticity impacts the shapes of marginal cost and thus the benefits of price adjustment, as explored in our robustness section 6.1.

The key parameters govern the price adjustment costs and the dispersion of idiosyncratic productivity. These are calibrated so that the steady state of our economy without aggregate shocks matches a set of moments.[20]

Even in the absence of aggregate shocks, the model produces a rich set of cross sectional moments given the presence of idiosyncratic shocks to productivity, idiosyncratic demand shocks and menu costs. The model calibration related to the distributions of the shocks rests on evidence related to the distributions of TFPQ and TFPR as well as the frequency of price adjustment.

The parameters characterizing the distribution of menu costs come directly from Dotsey and Wolman (2019) and are shown in the top panel of Table 1.[21] Note that this parameterization allows a free price adjustment with probability slightly over 5%. A period is a month.

The linear-quadratic specification leaves three parameters, $(\varepsilon, \sigma_z, \sigma_d)$ to be determined. To do so, we use moments from Vavra (2014) and Foster, Haltiwanger, and Syverson (2008) as shown in Table 2.

[19] Appendix 8.3 characterizes the SREE for the linear quadratic preferences.
[20] The steady state is just the SREE given above with the restriction that $x = 1$ with probability one.
[21] These are discussed in detail in Appendix 8.4.2.

Table 2: Matching Moments

Moment	Data	Model	Source
$disp_R$	0.102	0.103	Vavra (2014)
$disp_Q/disp_R$	1.181	1.181	Foster, Haltiwanger, and Syverson (2008)
$freq_{\Delta p}$	0.110	0.127	Vavra (2014)

Note: This table shows basic moments computed from time series averages and the steady state of our model using the parameters in Table 1.

The frequency of price adjustment are taken from Vavra (2014), where the model is calibrated on a monthly frequency. For Vavra (2014), the standard deviation of TFPR on a monthly frequency is set to match the annual measure from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). Vavra (2014) reports the standard deviation of the innovation, the persistence of the shock and the probability of a change in his Table III of calibrated parameters. In our model, all young sellers draw and shock from an ergodic distribution. Thus we infer the standard deviation of TFPR from the standard deviation of the innovation and the persistence reported by Vavra (2014).

Given our of focus on the distinction between TFPQ and TFPR, independent observations on these objects is quite informative. From Foster, Haltiwanger, and Syverson (2008) annual estimates, we take 1.181 as the ratio of the dispersion in TFPQ to the dispersion in TFPR. From experiments, it seems that this ratio is not influenced by time aggregation: simulating a higher frequency model and time aggregating preserves this ratio.[22]

As seen in Table 2, the calibration matches the moments well, though we do not quite reproduce the frequency of price adjustment reported in Vavra (2014).[23] The calibrated value of ε is below the level of other studies, such as Vavra (2014) and Golosov and Lucas (2007). Also, the dispersion of demand shocks is significantly lower than the dispersion of technology shocks, in contrast to Hottman, Redding, and Weinstein (2016) and Eslava and Haltiwanger (2020). We discuss the consequence of these differences in section 6.1 on alternative calibrations.

3.2 Seller Choices

This section illustrates the quantitative properties of the seller's choices for the linear-quadratic economy. Among other things, it makes clear that the policy functions from the overlapping generations model have properties quite similar to those produced by an infinitely lived seller. Throughout we focus on the response to idiosyncratic shocks, leaving aggregate shocks to the next section.

3.2.1 Pricing

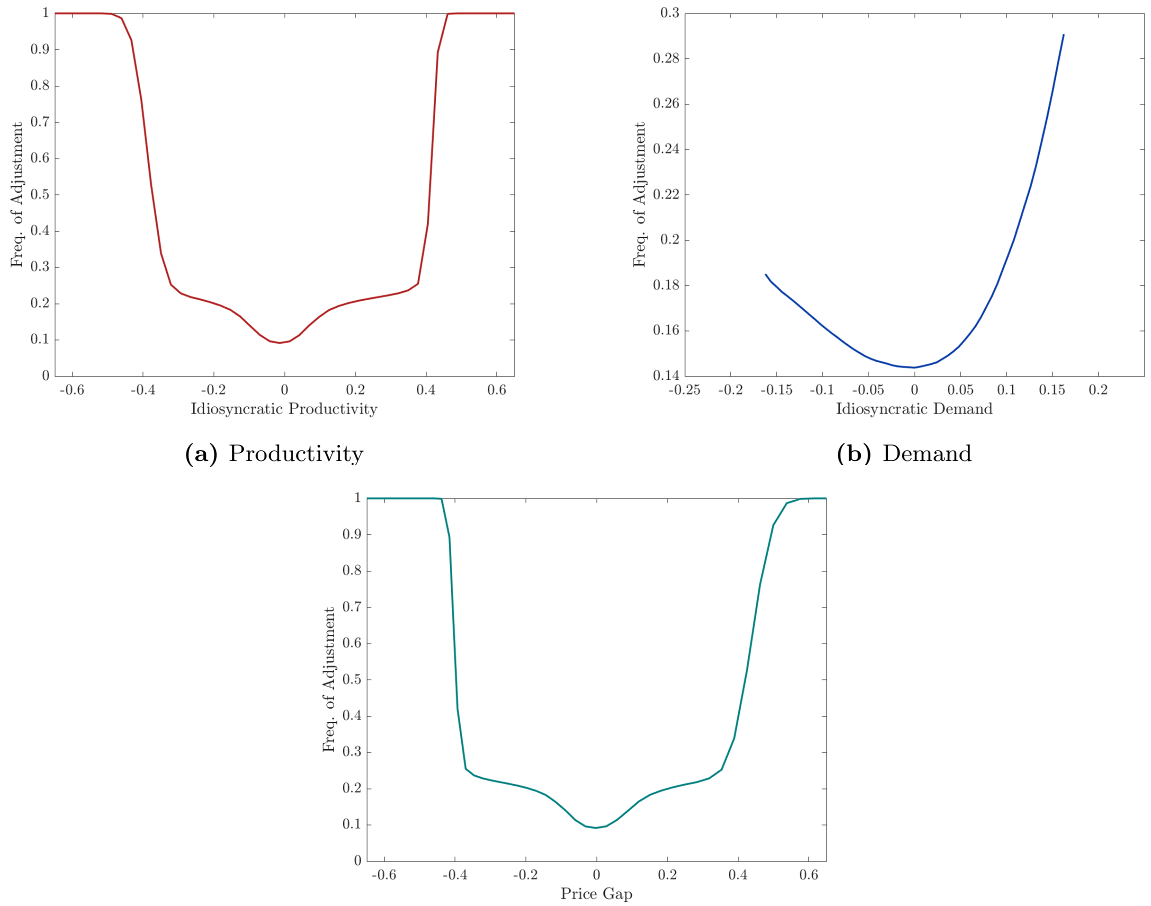
As in the traditional state dependent pricing model, prices are adjusted only for sufficiently large shocks and the region of adjustment depends on the adjustment costs. In addition, because of the presence of stochastic menu costs, the probability of adjustment, given z, lies strictly in $(0, 1)$ unless z is in one of the tails. These properties is illustrated in Figure 2 in the steady state of our model.

[22] This was studied in a partial equilibrium setting with Calvo price setting.

[23] One point of difference is that Vavra (2014) excludes temporary adjustments.

Two perspectives are shown in the figure. In the top two panels, the adjustment probability depends on the idiosyncratic productivity, on the left, and idiosyncratic demand, on the right. The adjustment probability is U-shaped indicating that adjustment is more likely for extreme values of these shocks. The adjustment rate depends on the parameters of the menu cost function as well as the elasticity of substitution, ε. For larger values of this elasticity, the substitution from other sellers is stronger, inducing more adjustment by a given seller.

Figure 2: Adjustment Probabilities



(a) Productivity **(b)** Demand

(c) Gap

Note: These figures show the adjustment rates of a seller in a steady state on idiosyncratic productivity, demand and the gap.

The bottom panel provides an alternative but equivalent expression of the adjustment probability. Here the horizontal axis measures the difference between the log of the price the seller would set if adjustment was free and the log of the *ex ante* price. This measure, often called the gap, is the foundation for the extensive research, from Caballero and Engel (1993) and Caballero and Engel (2007), on the relationship between adjustment rates and (price) gaps.[24] The likelihood of price adjustment as a function of the gap inherits the U-shaped patterns of the responses of adjustment to technology and demand shocks.

[24] This is used in Vavra (2014) too.

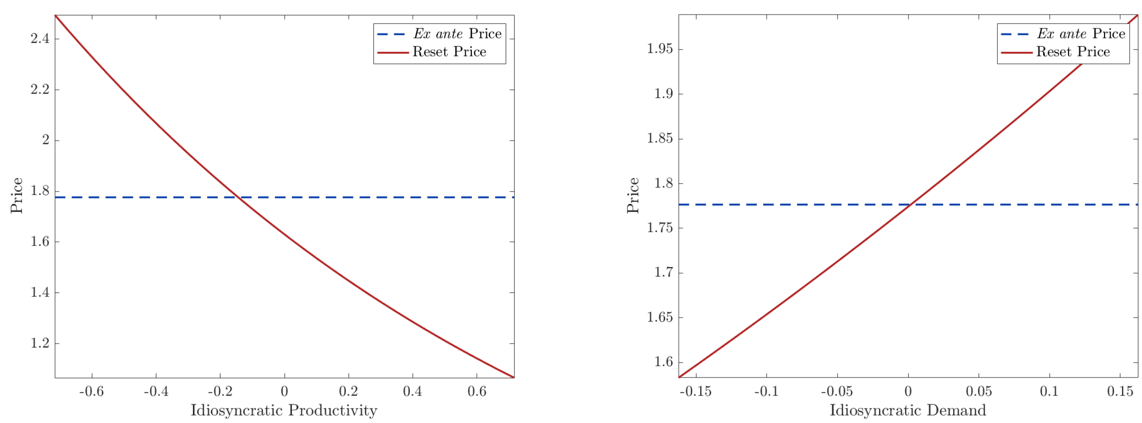
This is a natural metric for this analysis. In the overlapping generations model, the gap is not an approximation for the actual state but is a summary statistic for the gains to adjustment, to be weighted against the costs. That is, the structure of this model fits exactly with the requirements of the approach that summarizes the state through a price gap.[25]

As we shall see, the representation of adjustment rates as a function of the price gap is more convenient. Once aggregate shocks are introduced, the mapping from the idiosyncratic shocks to the likelihood of price adjustment will become state dependent. But, as made clear in Caballero and Engel (1993) and used as well in Vavra (2014), variations in idiosyncratic as well as aggregate states are neatly summarized by the price gap so that the adjustment probability is not a state dependent function of the price gap. Instead, the aggregate shocks impact the distribution of the price gaps across sellers. Interacting with the non-linear hazard, the distribution of these gaps will have aggregate implications.

The fact that the model economy produces this shape for the adjustment rate is important for two reasons. First, it confirms that state dependent pricing in the overlapping generations model produces patterns that are similar to other models. There is nothing special about the OG pricing structure with respect to the shape of this adjustment hazard.

Second, as the analysis develops, the aggregate economy will display non-monotonic responses to various types of shocks. Those patterns can be traced back to the U-shaped adjustment rate. Because the equilibrium of the model is characterized directly, that is without log-linear approximations, the aggregate non-linearities produced through this hazard will be sustained.

Figure 3: Reset Price



(a) Dependence on Productivity **(b)** Dependence on Demand

Note: These figures show the dependence of the reset price on idiosyncratic productivity and idiosyncratic demand.

Conditional on adjustment, the optimal price of the seller satisfies the first-order condition, (8), in the steady state where $x = 1$ with probability 1. In the calibrated model, the *ex post* optimal price is a decreasing function of productivity. This is shown in Figure 3. The dependence of this price on productivity will be impacted by the elasticity of demand.

[25]This is because of the limited time horizon. In an infinite horizon setting, the target price is often defined as the optimal price in the absence of adjustment costs assuming integrated shocks. Here no assumptions on the distribution of future variables are needed and, of course, permanent versus temporary opportunities to adjust are equivalent.

3.2.2 Output and Employment Responses

This subsection studies the employment and output response to idiosyncratic demand and productivity shocks. The results are enriched by the endogenous pricing decision of sellers.

Table 3 reports regression results estimated from simulated data for experiments characterized by the type of shocks: (i) idiosyncratic productivity shocks and, (ii) idiosyncratic demand shocks. The dependent variable is either the (log of) producer employment or output. The columns indicate the response of sellers who did and did not choose to adjust their price.

For the employment column, the negative coefficients for those sellers choosing not to adjust their price indicate the role of these rigidities on the employment response. The negative effective is present, though weaker, even when monetary shocks are in the model. For the adjusters, the effect of productivity on employment is always positive.

The same is true for the output of adjusters: output expands with either productivity or money shocks. For non-adjusters, idiosyncratic productivity shocks have no output effects since demand is given. But, if there is a positive idiosyncratic productivity shock then the sales of non-adjusters decline. Though nominal spending is held fixed, aggregate prices are lower so that a seller not adjusting its price has a high relative price and thus lower sales.

Table 3: Dependence of Employment and Output on Productivity and Demand

	Employment		**Output**	
	Adj.	*No Adj.*	*Adj.*	*No Adj.*
Productivity	0.235	-0.573	0.825	0
Demand	0.406	1.367	0.406	1.367

Note: This table shows the effects of idiosyncratic productivity (z) and demand (d) on producer-level employment and output conditioning on price adjustment status.

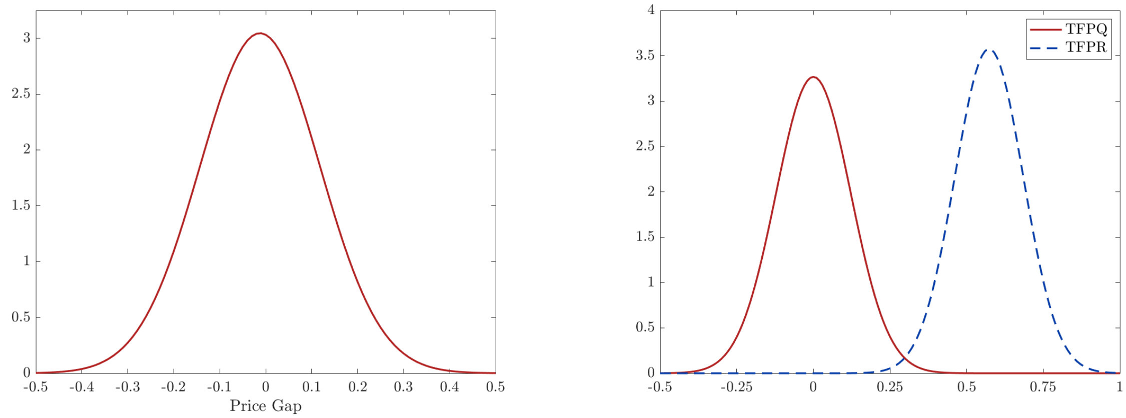
3.3 Aggregate Implications

In the absence of shocks, the only interesting aggregate features are the distributions of prices, the gaps and TFPR, given the distribution of TFPQ. The pricing itself has an extensive margin, to adjust or not, as well as an extensive margin regarding the response of the reset price to the idiosyncratic state z.

Figure 4a presents the steady state distribution of gaps. It is centered around zero and reflects the underlying distribution of the idiosyncratic productivity shocks. Clearly there are many sellers with relatively small gaps and who, from the adjustment hazard, are unlikely to adjust their price. Those in the tail have a larger gain to adjustment and thus are more likely to adjust.

Figure 4b shows the distributions of the two measures of productivity in the steady state. The distribution of TFPQ is given while that of TFPR comes from the interaction of the TFPQ distribution and the pricing choices of sellers. Since prices, contingent on resetting, are decreasing in productivity, there is less dispersion in TFPR than in TFPQ, as seen in Figure 4b.

Figure 4: Aggregate Implications



(a) Gap Distribution: Steady State

(b) Productivity Distributions: Steady State

Note: This figure shows the gap and productivity distributions in the steady state.

4 Cyclicality of TFPR Dispersion

The model of state dependent prices provides a basis to study the cyclicality of TFPR dispersion. As emphasized by Foster, Haltiwanger, and Syverson (2008), the measurement commonly taken from plant-level studies is TFPR not TFPQ. Output and revenue measures of productivity are not same and their distributions may covary in different ways over the business cycle. Theory exercises that equate TFPR with TFPQ miss the role of price in the mapping between these measures of productivity.

The question is whether the model of price setting can reproduce the countercyclical dispersion in TFPR seen in the data, as well as other pricing facts. This depends both on price setting behavior and exogenous variations. Here the exogenous variations include changes in the dispersion of idiosyncratic productivity, ($disp_Q$), changes in the dispersion of the idiosyncratic demand ($disp_D$), aggregate money shocks (x) and changes in the mean of TFPQ (μ_Q).

It is almost immediate that variations in the dispersion of TFPQ will cause variations in the dispersion of TFPR, incorporating optimal price setting. The issue here, as we shall see has to do with the cyclicality of these variations in dispersion. But what about the other shocks? They operate directly on the $disp_R$ given $disp_Q$. Is there any *ex ante* reason to believe they might lead to countercyclical dispersion in TFPR?

For this, consider the following decomposition of the variance in the log of TFPR (*tfpr*):

$$Var(tfpr) = Var(tfpq) + Var(ln(\text{p})) + 2 \times Cov(ln(\text{p}), tfpq). \quad (13)$$

This follows directly from the definition of TFPR: *tfpr*=ln(p)+ *tfpq*.

Table 4 shows this decomposition in the data and in the models, with the latter discussed later. The "FHS" row shows this decomposition for the data from Foster, Haltiwanger, and Syverson (2008). As noted earlier, the variance of *tfpr* is less than that of *tfpq*. This is a consequence of the negative covariance between prices and *tfpq*. So variations in the dispersion in *tfpr* can be created either by variations in the log of prices or through their covariance with *tfpq*.

The latter effect is directly related to the emphasis on pricing in this paper. As we shall see, prices adjustments are more frequent for extreme values of a money shock. Once the fixed cost of adjustment is

Table 4: Variance Decomposition: Data

	$Var(tfpr)$	$Var(tfpq)$	$Var(ln(\text{p}))$	$Cov(ln(\text{p}), tfpq)$
FHS	0.0484	0.0676	0.0324	-0.0258
recessions	0.0618	0.0676	0.0506	-0.0282

Note: This table shows the decomposition of the variance of *tfpr*. FHS data are annual (Foster, Haltiwanger, and Syverson (2008)). The percent changes for recession $var(tfpr)$ comes from RUBC, the recession $var(ln(\text{p}))$ is from Vavra (2014), the $cov(\cdot)$ is solved. Recessions are calculated assuming $disp_Q$ is fixed. All variables are logarithms.

paid, the seller can not only align the price to the nominal shock but also to the idiosyncratic component of *tfpq*. Thus, in this example, nominal shocks not only impact the dispersion of prices but also the covariance between prices and *tfpq*, affecting the dispersion of *tfpr*.

The second row labeled recessions is based upon but not taken directly from the data since the evidence in Foster, Haltiwanger, and Syverson (2008) does not have a cyclical component. It is constructed as a thought experiment where the increase in the variance of *tfpr* during a recession is taken from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). By assumption, the variance in *tfpq* is held fixed. The increased variance in p comes from Table 1 of Vavra (2014). The residual is the covariance of prices and *tfpq*, which is a key to generating cyclical variations in the dispersion of TFPR.

Leaving aside shocks to the dispersion in *tfpq*, the challenge is then to find exogenous variations that would create countercyclical dispersion *tfpr* through an increased dispersion of prices along with an increase in the (absolute) value of the covariance. From this exercise, it seems that shock(s) that can create both increased dispersion in prices as well as a higher (in absolute value) covariance of prices and idiosyncratic productivity can indeed increase the dispersion in measured TFPR. From our model, variations in these moments come both from the extensive margin of price adjustment as well as the dependence on productivity, conditional on adjustment.

4.1 Main Findings: a Preview

Table 5 summarizes our main findings and serves to organize the more detailed discussion that follows. It displays, by source of variation, the cyclical patterns of dispersion in *tfpr*, the dispersion of price changes, and the frequency of price adjustment.[26]

The table is discussed in detail in this section, first by looking at each shock independently. We then consider some shocks in tandem, as in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) and Vavra (2014). Finally, we allow the monetary authority to respond to variations in the mean and dispersion of TFPQ and study the implications for the dispersion of TFPR.

The results are best evaluated relative to moments from the data. **From various studies, the dispersion of *tfpr* is countercyclical, the dispersion of price changes and frequency of price changes are countercyclical.**[27]

[26] For this part of the analysis, a recession (expansion) refers to output below (above) its steady state value.

[27] The negative correlation of output (growth) and $disp_R$ comes from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). Kehrig (2011) finds that the correlation of (detrended) output and the dispersion of productivity is -0.293 for non-durables and -0.502 for durables, in Table 2. His Table 4 makes clear that the countercyclicality is robust to various output measures. Evidence on the dispersion and frequency of price changes comes from Vavra (2014), Table 4: the dispersion of price changes is higher in recessions as is the frequency of price changes.

By choice, we do not use correlations to summarize business cycle properties. The model, as suggested by the U-shaped hazard and discussed further below, has very non-linear responses to shocks. Looking at these through the lens of correlations can lead to the omission of the rich interactions produced by the model.

Table 5: Cyclical Variations

Shock	$disp_R$		$disp_{\Delta p}$		$freq_{\Delta p}$	
	Contraction	Expansion	Contraction	Expansion	Contraction	Expansion
			Baseline Parameterization			
$disp_Q$	0.047	0.131	0.020	0.167	0.065	0.294
x	0.088	0.095	0.125	0.114	0.278	0.221
$disp_D$	0.103	0.103	0.073	0.078	0.145	0.150
μ_Q	0.090	0.102	0.129	0.136	0.266	0.298
$disp_Q, \mu_Q$	0.126	0.057	0.208	0.082	0.328	0.164
			Leaning Against the Wind			
$disp_Q$	0.093	0.057	0.192	0.062	0.527	0.157
μ_Q	0.082	0.076	0.141	0.141	0.365	0.500

Note: This table shows the cyclical patterns of the dispersion in TFPR, $disp_R$, the dispersion in price changes, $disp_{\Delta p}$ and the frequency of price adjustment, $freq_{\Delta p}$.

4.2 Dispersion in TFPQ Shocks

The analysis of countercyclical variation in TFPR dispersion starts with an obvious hypothesis: variations in $disp_Q$ drive the cyclicality of $disp_R$. To order for this explanation to be consistent with data patterns, it must be that: (i) increased dispersion in TFPQ creates increased dispersion in TFPR and (ii) increased dispersion in TFPQ causes economic downturns. We demonstrate that the model does not produce these patterns: **variations in the dispersion of TFPQ do not generate countercyclical fluctuations in the dispersion of TFPR.**

Specifically, here we study the effects on $disp_R$ of an increase in $disp_Q$, modelled as a mean preserving spread in the distribution of z. To be clear, the effects highlighted here come from realized changes in the distribution of TFPQ, there is no uncertainty effect in the analysis.

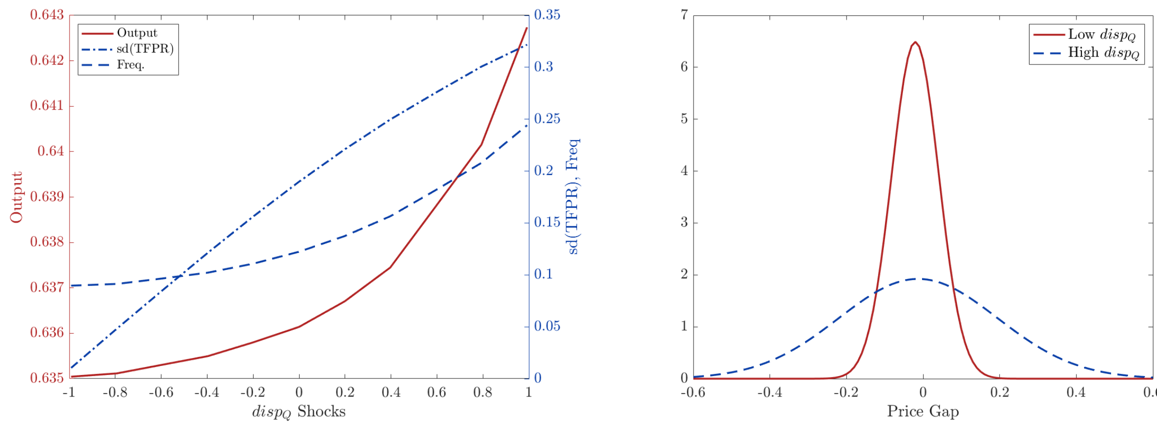
Variations in $disp_Q$ will impact $disp_R$ in two ways. First, of course, there is the direct effect: given prices, an increase in $disp_Q$ will translate into an increase in TFPR dispersion. Second, pricing behavior will adjust, potentially magnifying (reducing) the effects of the increase in $disp_Q$. The sign and size of this latter effect will depend on the properties of the revenue function and, as emphasized by our model, the pattern of price adjustment.

Figure 5a shows the response of output, the frequency of price adjustment and $disp_R$ in response to variations in $disp_Q$. Clearly output is an increasing function of this dispersion, allowing sellers with high productivity to expand.[28] The frequency of price adjustment itself increases as the increased dispersion in z puts more weight on the tails of the gap distribution, inducing more price adjustment. This is clearly evident in Figure 5b where the gap distribution is shown for two levels of $disp_Q$.

Overall, for this case, drawing on Figure 5a and Table 5, $disp_R$ is monotonically increasing in $disp_Q$ and hence in output. This is inconsistent with the motivating evidence. The findings about the cyclicality

[28] Importantly, these reallocation effects are hampered by both price rigidity and the immobility of labor.

Figure 5: $disp_Q$ Shock



(a) Responses **(b)** Gap

Note: This figure shows the relationship between output, $disp_R$ and the frequency of price adjustment as well as the price gaps as a function of shocks to $disp_Q$

of the dispersion in price changes and frequency are consistent with the findings of Vavra (2014) if $disp_R$ was countercyclical. **But the model is inconsistent with the data in terms of the motivating observation of countercyclical dispersion in TFPR.** Consequently, from Table 5, the variations in price change dispersion and frequency are counter to the data.

4.3 Money Shocks

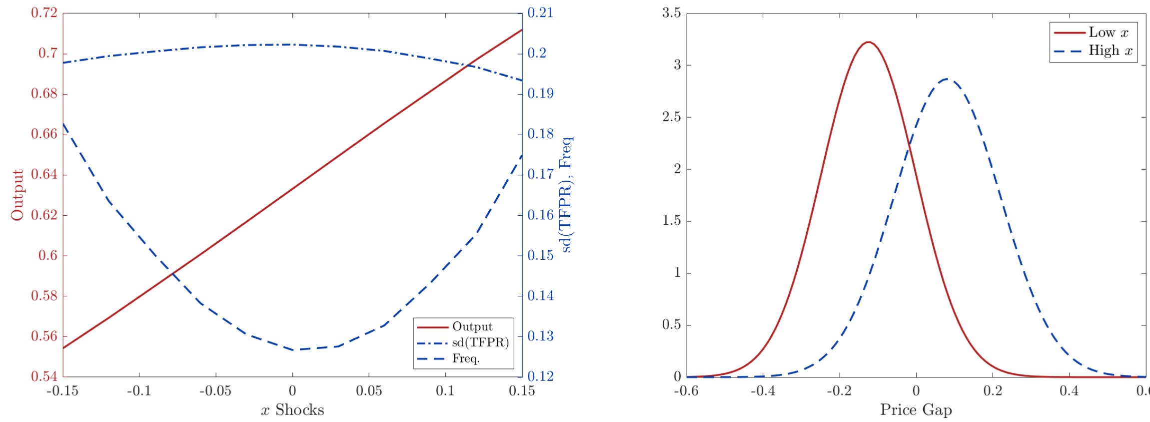
A second aggregate shock comes from monetary innovations, x. Due to price rigidities, monetary shocks impact real output. Further, the distribution of TFPR is impacted by monetary shocks, given the distribution of TFPQ, due to both the intensive and extensive margins of price adjustment.

From Table 5, for this source of variation, $disp_R$ is procyclical. As for the moments characterizing pricing, both the dispersion of price changes and the frequency of adjustment are countercyclical, in line with data patterns.

Figure 6a shows the response of output, $disp_R$ and the frequency of price adjustment to monetary shocks. Reflecting price rigidities, output is a monotone function of the innovation to the money supply. As highlighted in Figure 2, the frequency of adjustment is a U-shaped function of the gap. That is reflected here in the U-shaped frequency of adjustment in response to the money shocks. Importantly for our analysis, this translates into an inverse U-shaped relationship between $disp_R$ and the money shock. At extreme values of the shock, price adjustment is much higher. Since price setters are responding to the common realization of x, there is a reduction in the dispersion of TFPR. Though the realized idiosyncratic productivity, z, is independent of x, the selection into adjustment, again using Figure 2, will be those in the tails of the productivity distribution.

The effects of the money shock on the gap distribution is shown in Figure 6b. In contrast to the increased dispersion of the gap distribution from a $disp_Q$ shock, the monetary shock causes a rightward shift. The additional weight on the right tail from a high value of x will increase the frequency of upward price adjustments.

Figure 6: Money Shock

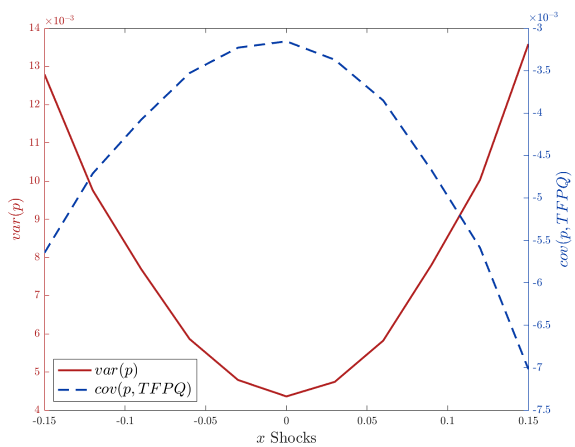


(a) Responses **(b)** Gap

Note: This figure shows the relationship between output, $disp_R$, the frequency of price adjustment and the gap as a function of money shocks.

It is useful to understand how a money shock influences the distribution of TFPR. The mechanism is illustrated in Figure 7 which shows two cross-sectional moments, the variability of prices and the covariance of prices and productivity, vary with the money shock. With $disp_Q$ fixed, the distribution of TFPR will depend on the pricing choice of sellers. From this figure, for extreme values of the money shock, the standard deviation of prices is higher and the covariance of price and productivity is higher in absolute value. This reflects the increased frequency of adjustment, as in Figure 6a, as well as the dependence of prices on z for those sellers who choose to reset. This is in keeping with the role of dispersion and covariance brought out in Table 4.

Figure 7: Price Dispersion and Covariance of Prices and Productivity

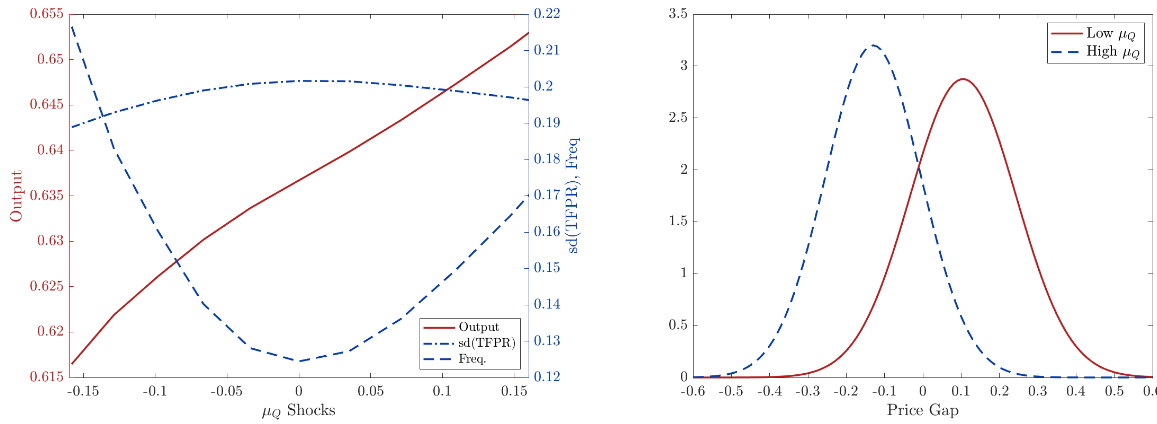


Note: This figure shows response of the variance of price, the standard deviation of prices and the covariance of prices and productivity at the micro-level to money shocks.

Overall, as real output increases with the money shock, the model implies that the standard deviation of TFPR is not a monotone function of economic activity when fluctuations are induced by money shocks. It can be lower in recessions and also lower in expansions when the money shocks take relatively extreme values. **Thus, the model can produce countercyclical dispersion in TFPR, for a given distribution of TFPQ, when money shocks are surprisingly large**. Importantly, the change in the dispersion of prices and their covariance seen in this experiment follows the qualitative pattern of the data, as shown in Table 4.

4.4 Shocks to the Mean of TFPQ

Figure 8: μ_Q Shock



(a) Responses **(b)** Gap

Note: This figure shows the relationship between output, $disp_R$, the frequency of price adjustment and the gap as a function of shocks to the mean of TFPQ.

The another leading source of variation is the more standard shock to the average productivity, i.e. the mean of TFPQ, denoted μ_Q. As before, the interest is in the cyclicality of the dispersion in TFPR induced by this shock. For now, we study its impact in isolation. Experiments below couple this with a shock to $disp_Q$ as well as a monetary response.

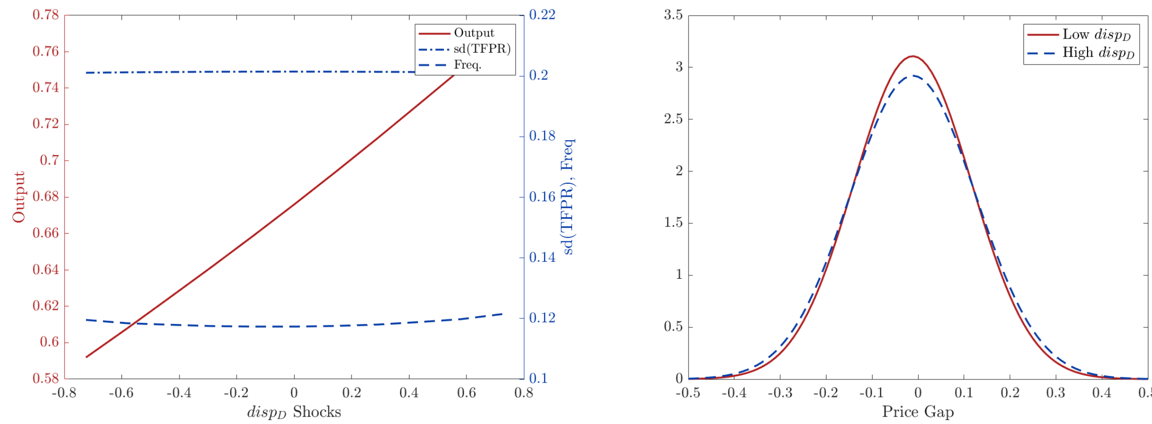
Figure 8a summarizes the findings. As in standard RBC models, output is an increasing function of mean productivity. The frequency of price adjustment is again U-shaped, reflecting the larger gains to adjust for more extreme realizations of mu_Q along with the shift in the gap distribution, shown in Figure 8b. The dispersion in TFPR is almost flat, decreasing slightly for realizations in the tails where there is more price adjustment.

Thus this case does not produce the data pattern of countercyclical dispersion in TFPR. Further, from Table 5, for this source of variation, both the dispersion of price changes and the frequency of adjustment are procyclical, in contrast with data patterns. But, as discussed below, this source of variation is of more interest when combined with a shock to $disp_Q$.

4.5 Dispersion of Demand Shocks

A final source of aggregate variations arises from changes in the dispersion of idiosyncratic demand shocks. As with variations in $disp_Q$, this is a mean preserving spread in demand shocks. The money shocks can be interpreted as variations in the mean of demand.

Figure 9: $disp_D$ Shock



(a) Responses **(b)** Gap

Note: This figure shows the relationship between output, $disp_R$ and the frequency of price adjustment as a function of shocks to $disp_D$.

As shown in Figure 9a, output increases with demand dispersion, as it did with increased dispersion in TFPQ. In response to increased dispersion in demand shocks, $disp_R$ is slightly countercyclical. This is quite different than the response of $disp_R$ to an increase in $disp_Q$. Part of the explanation lies in the response of output and employment to a demand shock at the producer level, summarized in Table 3. From Table 5, the frequency of price adjustment and its dispersion increase with this shock, so that both of these moments are procyclical.

4.6 Shocks to the Dispersion and Mean of TFPQ

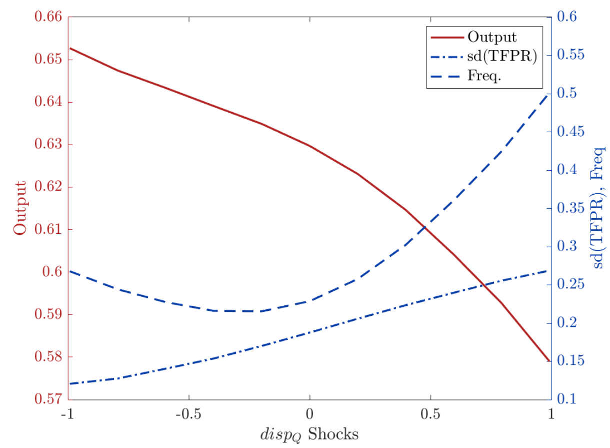
In many studies, such as Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) and Vavra (2014) the shock to dispersion and to the mean of TFPQ are studied jointly. Given the prominence of this case in the literature, it is important to study this case in detail. Here we follow the baseline model in Vavra (2014) and assume the shocks are perfectly negatively correlated: $corr(disp_Q, \mu_Q) = -1$.

From Table 5, this is the experiment that brings the model and data patterns closest together. All three moments, $disp_R$, price change the dispersion and frequency of adjustment are countercyclical.

The results from Figure 10 illustrates the effect of combining these shocks. The horizontal axis shows $disp_Q$. By construction, as it increases μ_Q falls. From the graph, this latter effect dominates so that output contracts despite the increase in dispersion. But the dispersion in TFPR increases, albeit modestly, driven by the increase in $disp_Q$.

Still, the moments are not monotone. For example, the frequency of price adjustment, due to the U-shaped hazard, is also relatively high for low values of $disp_Q$ when output is high. We return to these nonlinearities below.

Figure 10: Combined $disp_Q, \mu_Q$ Shocks



Note: This figure shows the relationship between output, $disp_R$ and the frequency of price adjustment as a function of shocks to the $disp_Q$. By construction, as the dispersion increases, the mean of TFPQ falls.

5 Monetary Feedback Rules

One important theme of the analysis is the nonlinearity produced by the U-shaped frequency of price adjustment. This was shown to matter in the response of the economy to money shocks x. Building on this, we enrich the setting to allow interactions between the shocks, focusing on monetary policy responses. As we see, allowing the monetary authority to link the distribution of x to the aggregate state can alter the cyclicality of $disp_R$. In this way, the implications of the model can be brought closer to some features of the data.

Specifically, suppose that the evolution of the money supply is given by:

$$M_{t+1} = M_t x_{t+1} = M_t[\Phi(s_{t+1}) + \tilde{x}_{t+1}]. \tag{14}$$

In this specification, the money stock follows the same stochastic process as above, with x_{t+1} representing the period $t+1$ money shock that is not predictable given period t information. But here, the growth of the money supply, $[\Phi(s_{t+1}) + \tilde{x}_{t+1}]$ has two components. The first is the feedback rule where $\Phi(s_{t+1})$ allows money growth to depend on the period $t+1$ state of the economy. The second is the money shock, denoted $\tilde{x}_{t+1}$ above.

We focus on two specific cases, distinguished by the source of fluctuations in the aggregate economy. These cases produced variations in the dispersion of TFPR that are qualitatively similar to data moments.[29]

In the first, the monetary authority responds to changes in the dispersion of TFPQ. Let μ_{disp_Q} be the average value of $disp_Q$ and consider

$$\Phi(disp_Q) = \zeta \times (disp_Q - \mu_{disp_Q}). \tag{15}$$

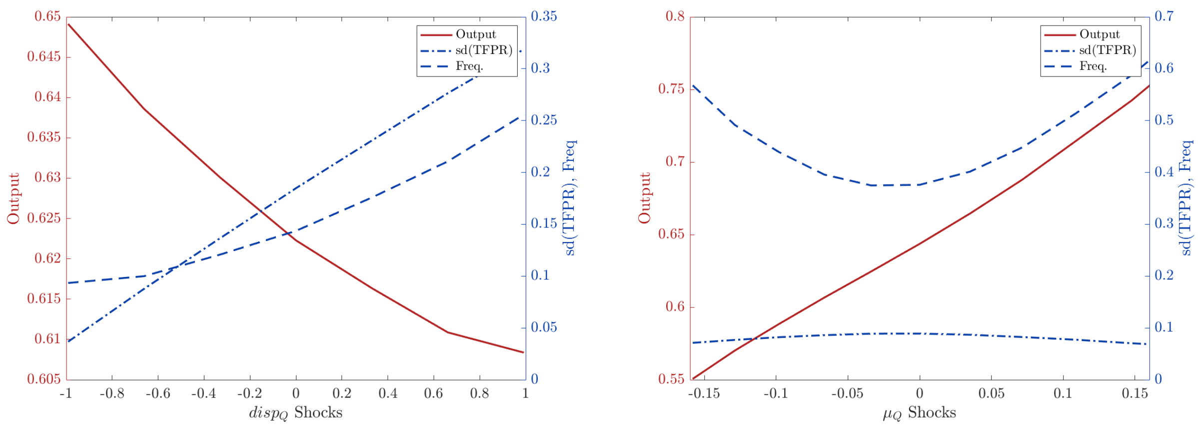
In a similar fashion, let μ_{μ_Q} be the average value of the mean of TFPQ and consider

$$\Phi(\mu_Q) = \zeta \times (\mu_{\mu_Q} - \mu_Q). \tag{16}$$

[29]This was not the case for monetary policy interacting with $disp_D$ shocks.

In both formulations, the feedback is characterized by a single parameter, ζ.

Figure 11: Monetary Responses



(a) $disp_Q$ Shocks **(b)** μ_Q Shocks

Note: This figure shows the effects of a response in monetary policy to $disp_Q$ and μ_Q shocks.

Given a monetary feedback rule, it is straightforward to extend the analysis of a SREE from Appendix 8.3 to include (14). Note that the monetary feedback rule impacts agents both as young price setters and as old agents, both in terms of the distribution of the stochastic transfer and the equilibrium prices they face as buyers. As in the previous analysis, all of the newly created money is distributed as a proportional transfer. But in this specification, it is feasible for the monetary authority to link these transfers to the current state of the economy. If prices were perfectly flexible, there would be no real effects of this monetary policy. Further, since private agents share the information of the monetary authority, there is no information transmitted to the private sector by this policy.

The SREE was characterized for both shocks to μ_Q and $disp_Q$. The formulation that created countercyclical dispersion in TFPR had $\zeta < 0$ so that the monetary authority was "leaning against the wind".

Consider first the results when the economy is driven by variations in $disp_Q$, along with money shocks. Figure 11a illustrates the outcome and Table 5 summarizes the moments.

In this case, the patterns in the simulated data match those in the actual data. A feedback rule with $\zeta = -0.05$ generates countercyclical dispersion in $disp_R$. With this policy, the monetary authority responds to higher than average dispersion in idiosyncratic profitability shocks by reducing the average growth of the money supply. In the absence of the intervention, output would be positively correlated with $disp_Q$. So, the monetary authority appears to be leaning against the wind. But in this case, the response to the policy outweighs the direct effect of $disp_Q$ so that increased dispersion in z is associated with an economic downturn. The dispersion in TFPR follows that of TFPQ, so that $disp_R$ is countercyclical. Note that this result does not occur without monetary feedback. As noted earlier, with $\zeta = 0$ the model creates procyclical dispersion in TFPR.

From Table 5, the frequency of price adjustment is higher in the recession. Further, the dispersion of price changes is also higher in recessions. These patterns match those in the data.

A second interesting case arises from the response of the monetary authority to mu_Q shocks. This is illustrated in Figure 11b. In this setting, the nonlinear response of price setting to the state is important.

Despite the monetary authority leaning against the wind, output increases with the mean of TFPQ. But, in contrast to the case with no monetary feedback in Figure 8a, now $disp_R$ varies considerably with the aggregate productivity shock. This is because of the response of price setters interacting with the money shock.

From Table 5, $disp_R$ is countercyclical, but not nearly as much as in the case of monetary feedback to variations in $disp_Q$. A difference with the data appear in the pricing moments. In particular, the frequency of adjustment is procyclical, produced by the asymmetry in the U-shaped hazard from Figure 8a.

6 Additional Properties

This section looks at additional properties of the model. The presentation starts with a discussion of the sensitivity of moments to parameters. The second point highlights the nonlinearities of the economy by presenting correlations conditional on the output gap. The third uses the model to understand the findings of Tenreyro and Thwaites (2016) concerning the nonlinear effects of monetary policy. The final part introduces uncertainty and, as in Vavra (2014), finds no role for it.

6.1 Sensitivity to Calibration

Here we study the mapping from parameters to moments. There are two key parameters: (i) the elasticity of substitution between products, ε, and (ii) the convexity in the disutility of work, $g(n) = \frac{n^\phi}{\phi}$. The point is to understand how the shapes of these functions impact the results in Table 5.[30] In particular, we increase ε from its baseline value of 2.36 to 4 and decrease ϕ from 2 to 1.5, thus reducing the slope of marginal cost. Finally, we look at how the results depend on the relative magnitudes of technology and demand shocks. For the baseline, σ_d was about 18% of σ_z. For the robustness exercise, we set $\sigma_d = \sigma_z$, close to the findings reported in Eslava and Haltiwanger (2020), with σ_z remaining at its baseline value.

Table 6: Cyclical Variations: Robustness

Case	$disp_R$		$disp_{\Delta p}$		$freq_{\Delta}p$	
	Contraction	Expansion	Contraction	Expansion	Contraction	Expansion
Baseline	0.088	0.095	0.125	0.114	0.278	0.221
$\varepsilon = 4$	0.086	0.098	0.086	0.069	0.323	0.264
$\phi = 1.5$	0.120	0.117	0.116	0.118	0.094	0.094
$\sigma_d = \sigma_z$	0.577	0.582	1.024	1.215	0.875	0.776

Note: This table shows cyclical patterns for alternative parameters (rows) with fluctuations driven by monetary shocks.

For these experiments we focus on the case of money shocks alone. This allows us to isolate the effects of these parameter variations most succinctly. Table 6 summarizes our findings. The rows indicate the parameters that have been altered relative to the baseline, reproduced as the first row of each block. Each entry represents a simulation setting all other parameters at baseline levels.

[30] Vavra (2014) has linear disutility of work and an elasticity of substitution of 6.8. Golosov and Lucas (2007) also have linear disutility with an elasticity of substitution of between 6 and 10. They also include money in the utility function.

As indicated in the first row of Table 6, the model with money shocks alone did not produce countercyclical dispersion in TFPR. The property, inconsistent with the data, is retained for the alternative parameterizations except for the experiment with less curvature in the disutility of work.

With a higher value of ε relative to the baseline, the frequency of price adjustment is higher. As goods are more substitutable, the incentive to adjust prices when others do is higher. The dispersion in price changes is lower since the optimal price is driven more by the aggregate money shock compared to the idiosyncratic shock.

With $\phi = 1.5$, the moments are essentially acyclical. Notice that the frequency of price adjustment is much lower than the baseline since the cost of meeting variations in demand induced by money shocks is considerably lower. Accordingly, $disp_R$ is also much higher that in the baseline since variations in z are not being offset by price adjustments.

Increasing the dispersion of demand relative to productivity shocks has a large impact on these moments. With this increased source of variability in demand, the frequency of price adjustment is almost four times that of the baseline. The dispersion of price changes is also much larger. This alternative parameterization though does not bring the model with monetary shocks alone closer to the data.

6.2 Nonlinearities

As noted earlier, correlations have not been used to summarize model properties given the inherent nonlinearities created by the price adjustment hazard. One way to highlight this is to compute correlations conditional on the business cycle, measured by the difference between output and its steady state value.

Table 7 presents correlations of key variables with output conditional whether output is above (expansion) or below (contraction) its stationary level. This is shown for the various shocks, including the monetary feedback rules.

First, looking at the monetary shock case, the frequency of price adjustment is negatively correlated with output in a contraction and positively correlated with output in an expansion, -0.648 and 0.977 respectively. This is a direct consequence of the U-shaped hazard, as in Figure 6a. So when the money shock is above average, so is output. Within this region, higher realizations of the money shock increase the frequency of price adjustment and, at the same time, output expands. But, for values of the money shock below the mean (so that output is below its mean), the opposite occurs. For progressively lower values of x, again the frequency of price adjustment rises but output falls, producing a negative correlation in this region.

The dispersion of TFPR has an inverted U shaped in Figure 6a. This produces a negative correlation with output in expansions as $x > 0$. But the correlation switches sign when x is below its mean.

Second, note that in many cases other than money shocks, the correlations change sign with the state of the economy. This pattern of a positive (negative) correlation of price adjustment and output in expansions (contractions) is seen in the other cases except for $disp_Q$ shocks. In that case, the frequency of price adjustment is higher in expansions but, from Table 7, the correlation with output is negative, conditional on being in an expansion. And for some experiments, such as the μ_Q shock the correlations are quite high, conditional on the state.

Third, variations in shocks to x, $disp_D$ or μ_Q can each produce countercyclial $disp_R$ but only during expansions. The combination of $disp_Q, \mu_Q$ shocks generate this negative correlation in all states of the business cycle.

To be clear, these nonlinearities are a direct consequence of the U-shaped hazard. As that is a central feature of state dependent pricing models, these properties are not peculiar to our specification.

Table 7: Cyclical Variations: Conditional Correlations

Shock	$disp_R$		$disp_{\Delta p}$		$freq_{\Delta}p$	
	Contraction	Expansion	Contraction	Expansion	Contraction	Expansion
			Baseline Parameterization			
x	0.768	-0.959	-0.779	0.989	-0.648	0.977
$disp_Q$	0.685	0.306	0.586	-0.973	0.559	-0.965
$disp_D$	0.371	-0.163	0.281	0.425	-0.228	0.226
μ_Q	0.930	-0.941	-0.983	0.961	-0.957	0.921
$disp_Q, \mu_Q$	-0.488	-0.730	-0.967	0.551	-0.986	0.850
			Leaning Against the Wind			
$disp_Q$	-0.149	-0.257	-0.079	-0.420	0.344	-0.491
μ_Q	-0.051	-0.357	0.065	0.028	0.021	0.425

Note: This table shows the cyclical patterns of the dispersion in TFPR, $disp_R$, the dispersion in price changes, $disp_{\Delta p}$ and the frequency of price adjustment, $freq_{\Delta p}$. Here contractions and expansions are defined in levels relative to steady state.

6.3 Nonlinear Effects of Monetary Policy

Tenreyro and Thwaites (2016) argue output is less responsive to monetary policy during recessions. It is critical to note that by a recession they are referring to negative growth in output and not a level of output below trend. In fact, they find that the response of output to money shocks does **not** depend on the level of output relative to trend.[31]

We use our model, with its explicit distinction between TFPQ and TFPR, to study the state dependent effects of monetary shocks. The question is whether the model provides support for the findings of Tenreyro and Thwaites (2016).

To some extent, Vavra (2014) addresses this question. He argues that shocks to nominal spending will have a smaller effect on output when the dispersion of firm level productivity is higher, i.e. in a recession. This comes from his finding that the dispersion and frequency of price changes is countercyclical. Hence, recessions are associated with more frequent price adjustment and thus a smaller impact of monetary policy. But, as noted earlier, the focus was on the implications of countercyclical dispersion of TFPQ rather than the countercyclical dispersion in TFPR.

Table 8 quantifies the interaction between the state of economic activity and the response of output to a monetary innovation. It does so by regressing (log) real output on the (log) monetary shock, distinguishing recessions from expansions in two ways. The first row, labeled "Relative to Trend", views a recession as output below trend and an expansion as output above trend. The second, labeled "Growth" defines contractions (expansions) as three consecutive periods of negative (positive) output growth, in line with the dating policies of the NBER.

This experiment is built upon the model with shocks to $(disp_Q, \mu_Q)$ jointly, as in section 4.6. As presented, this combination of shocks matches the key data patterns. So it is of interest to add the monetary variations to them. In addition, there are orthogonal shocks to x and we assess the impact of those shocks on aggregate fluctuations.

[31] In fact, the baseline estimates of Tenreyro and Thwaites (2016) use a 7 quarter moving average of GDP growth to construct their indicator.

From the table, the response of output to a monetary innovation is larger during expansions compared to recessions. The mechanism is best understood from Figure 10. A recessions is associated with a large value of $disp_Q$ coupled with a reduction in mu_Q. From the figure, during the recession the frequency of price adjustment is higher. With more price flexibility in a recession, the real effect of the money shock is reduced. Therefore we see this countercyclical effectiveness of monetary shocks.

Note that this effect is consistent with the findings of Tenreyro and Thwaites (2016) only for the treatment where the aggregate state is determined by the growth in output. Further, as stressed in Tenreyro and Thwaites (2016), durable goods play a role in this asymmetry and are absent from this model.

Table 8: Regression of output on monetary shock depending on state

	Contraction	Expansion
Relative to Trend	0.578	0.644
	(0.001)	(0.001)
Output Growth	0.589	0.611
	(0.001)	(0.002)

Note: In the regression, monetary shock is interacted with indicator variable based on the status of the economy (contraction or expansion). This table reports the coefficient of these interaction terms. Standard errors are in parentheses.

6.4 Effects of Uncertainty

The distinction between uncertainty and dispersion is often blurred. The main effect of uncertainty, again expressed in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), is to create an incentive to wait and allow the uncertainty to be resolved. To the extent this leads to a decrease in spending, largely on durables, the uncertainty can be recessionary. This is often quite different from the positive effects of dispersion which can lead to an expansion in output, as discussed above.

The previous discussion highlighted the effects of dispersion on the frequency of price adjustment and thus the real effects of monetary shocks. Here we focus on how *ex ante* price and *ex post* respond to uncertainty over a distribution, not the realization of that change.

Our analysis includes distributions over four dimensions: (i) idiosyncratic productivity, (ii) idiosyncratic demand, (iii) money shocks, and (iv) aggregate productivity. Thus in principle one can study the effects of uncertainty with respect to each of these four distributions.

To do so, it is natural to create a Markov switching process for the dispersion of, say, idiosyncratic productivity. Price setters in period t would know the distribution of these shocks last period but in setting their *ex ante* price, the period t distribution, as well as that for period $t+1$ would not be known. Further, for those who adjust *ex post*, the uncertainty would remain over the distribution in the following period when they are consumers.[32] This is the nature of the uncertainty.

One extreme version of this Markow switching process is for the dispersion to be permanently high (low). **For the price setting problem of young agents, the *ex ante* price is essentially the same with**

[32]Thus the expectation on the left side of (24) is extended to include the conditional expectation over the future dispersion.

high dispersion of the idiosyncratic productivity shock as it is for the low dispersion case. In fact, this is true when the uncertainty is over the money transfer or the aggregate productivity distributions.

Given this, it is unlikely that *ex ante* uncertainty matters for the price setting problem. This is verified explicitly for the case of uncertainty over idiosyncratic productivity. Even if there is a positive probability of a regime shift in the distribution of z, the *ex ante* price is essentially unchanged.

This is an important finding. It makes clear that the effects come from dispersion not uncertainty. This is consistent with Berger, Dew-Becker, and Giglio (2020) who argue, at least for aggregate shocks, that uncertainty *per se*, had a negligible effect on real activity.

7 Conclusion

The analysis characterizes the properties of the distribution of TFPR in a stationary rational expectations equilibrium of a monetary economy with state dependent pricing. A quantitative version of the model is used to determine the cyclicality of the dispersion in TFPR as well as other key pricing moments, the cyclicality of both the frequency of price changes and their dispersion. This is studied by determining pricing decisions and thus the distribution of TFPR in the face of aggregate shocks to: (i) the money supply, (ii) the dispersion of TFPQ, (iii) the mean of TFPQ and (iv) the dispersion of demand. These are very conventional shocks for an aggregate economy, with recent attention given to variations in the dispersion of TFPQ and demand.

The moments are generated from a stationary rational expectations equilibrium without the need for linearization. This matters as the firm-level non-linearities in the state dependent pricing model carry over to the aggregate economy.

Looking at these shocks alone as well as combinations and allowing monetary feedback, there are a few cases in which the data patterns of countercyclicality in the dispersion of TFPR, the frequency of price adjustment and dispersion in price changes are matched. One case arises when there are negatively correlated shocks to the mean and dispersion of TFPQ. This combination was highlighted in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) to match aggregate fluctuations. Here the combination actually creates the countercyclical dispersion in TFPR assumed in that paper. Also, a monetary authority that leans against the wind in face of shocks to the dispersion of TFPQ creates an equilibrium that matches data patterns.

Admittedly these results are suggestive rather than definitive. The OG model, with only one period of price setting, misses some of the forward looking aspect of price adjustment. But, as argued in the text, the pricing behaviour in the model is similar to that produced by other state dependent pricing models. On the data side, it would be desirable to have higher frequency observations on both prices and quantities upon which to base a structural estimation exercise.

Throughout these exercises, one theme emerges: non-linearities in the response of the economy to monetary and dispersion shocks. Regardless of the source of aggregate fluctuations, the dispersion of TFPR is generally lowest for extremely low and high realizations and highest for the average state. This property of the model, driven by the U-shaped response of the frequency of price changes to money surprises, makes it useful to study the impact of monetary and productivity shocks using non-linear statistical models.

This suggests empirical exercises that goes beyond the traditional focus on correlations, say between output and the dispersion of TFPR. Tenreyro and Thwaites (2016) exemplifies this approach. There is certain value in looking further at price adjustment frequency as well as employment and output responses, at both the firm and aggregate levels, in a non-linear setting. For this, high frequency data on prices, output and employment is needed.

Finally, the model is used to study the effects of uncertainty on pricing. It seems clear that the effects highlighted in our analysis stem from dispersion not uncertainty. One interesting extension of our model would be to include some of the adjustment cost structure that creates a real options effect, as in Bloom (2009), **coupled with** state dependent pricing.

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8 Appendix

8.1 Idiosyncratic Demand Shocks

In the presence of idiosyncratic demand shocks, consider a consumption aggregator of

$$c = \left(\sum_i \alpha_i c_i^\gamma \right)^{\frac{1}{\gamma}}$$

with $\gamma \equiv \frac{\varepsilon-1}{\varepsilon}$. In this specification, α_i is a weight on good i as not all goods are weighted equally in utility. Relative demands are given by

$$\frac{c_i}{c_j} = \left(\frac{\alpha_j p_i}{\alpha_i p_j} \right)^{-\sigma}$$

where $\sigma = \frac{1}{\gamma-1}$. Define $\tilde{P} = (\sum_i \tilde{p}_i^{1-\sigma} \alpha_i)^{1/(1-\sigma)}$, so that

$$c_j = (\frac{\tilde{p}_j}{\tilde{P}})^{-\sigma} \frac{M}{\tilde{P}} = \alpha_j^\sigma (\frac{p_j}{\tilde{P}})^{-\sigma} \frac{M}{\tilde{P}}. \tag{17}$$

Here M is nominal spending and $\tilde{p_j} \equiv \frac{p_j}{\alpha_j}$.

We introduce relative demand shocks through this specification. For a given distribution of weights, there is nothing stochastic about the household problem with respect to tastes. That is, the young agent of generation t has fixed preferences of consumption goods when they are old. So the household problem specified in the main text remains, with the modified aggregator.

But, from the perspective of a seller, the model introduces uncertainty in that *ex ante* the seller does not know the taste shock pertaining to the particular good of that seller. This allows uncertainty in demand to exist from the perspective of a seller but not the consumer. The uncertain demand can impact the *ex ante* price as well as the *ex post* decision to adjust and, conditional on adjustment, the *ex post* price.

This specification leads to two types of shocks. First, there are seller specific realizations of demand shocks, denoted α, which directly impact revenues. Second there are variations in the distribution of α are studied through mean preserving spreads, denoted $disp_D$.

8.2 Generalized Definition of SREE

Here the definition of a stationary rational expectations equilibrium is generalized to include shocks to the distribution of plant-level productivity through μ_Q and $disp_Q$ as well as shocks to idiosyncratic demand, α and the distribution of the demand shocks, $disp_D$. Let $S = (x, \mu_Q, disp_Q, disp_D)$ be the aggregate state and $s = (z, \alpha, F)$ be the idiosyncratic state.[33] As earlier, M is the previous money stock and thus is known at the time prices are chosen *ex ante*.

A SREE is a set of price functions $(\bar{p}(M), \tilde{p}(M, S, s), P(M, S))$, value functions $(W^n(M, S, s), W^a(M, S, s))$, and a critical value of the price adjustment cost, $F^*(M, S, s)$ satisfying: (i) individual optimization by young price setters and old consumers, (ii) market clearing and (iii) consistency of beliefs and expectations for all states. These conditions can be written:

- $\bar{p}(M)$ solves the *ex ante* pricing problem given the state dependent price index $P(M, S)$;

$$\bar{p}(M) = argmax_p E_{S,s,S'} \left\{ V\left(\frac{R\left(M; p, \alpha; P(M, S), x\right) x'}{P(Mx, S')} \right) - g\left(\frac{d(M; p, \alpha; P(M, S), x)}{\mu_Q z} \right) \right\} \tag{18}$$

  for all M.

- $\tilde{p}(M, S, s)$ solves the *ex post* pricing problem:

$$\tilde{p}(M, S, s) = argmax_p E_{S'} \left\{ V\left(\frac{R(M; p, \alpha; P(M, S), x)x'}{P(Mx, S')} \right) \right\} - g\left(\frac{d(M; p, \alpha; P(M, S), x)}{\mu_Q z} \right). \tag{19}$$

  given $P(M, S)$, for all (M, S, s);

- At the critical adjustment cost, $F^*(M, S, s)$, the seller is just indifferent between adjusting and not:

$$F^*(M, S, s) \equiv W^n(M, S, s) - W^a(M, S, s)$$

[33]So here the notation is different from that in the text to be more explicit about aggregate and idiosyncratic variables.

for all (M, S, s), with $W^a(M, S, s)$ given by:

$$W^a(M,S,s) = E_{S'}\left\{V\left(\frac{R(M;\tilde{p}(M,S,s),\alpha;P(M,S),x)x'}{P(Mx,S')}\right)\right\} - g\left(\frac{d(M;\tilde{p}(M,S,s),\alpha;P(M,S),x)}{\mu_Q z}\right), \tag{20}$$

and $W^n(M, S, s)$ given by

$$W^n(M,S,s) = E_{S'}\left\{V\left(\frac{R(M;\bar{p}(M),\alpha;P(M,S),x)x'}{P(Mx,S')}\right)\right\} - g\left(\frac{d(M;\bar{p}(M),\alpha;P(M,S),x)}{\mu_Q z}\right). \tag{21}$$

- $P(M, S)$ is the aggregate price index in state (M, S) given by:

$$P(M,S) = [E_s(1-\Omega(F^*(M,S,s)))\bar{p}(M)^{1-\varepsilon} + E_s(\Omega(F^*(M,S,s))\tilde{p}(M,S,s)^{1-\varepsilon})]^{\frac{1}{1-\varepsilon}} \tag{22}$$

where $d(M;\bar{p}(M),\alpha;P(M,S),x) = \alpha^\varepsilon\left(\frac{\bar{p}(M)}{P(M,S)}\right)^{-\varepsilon} Y$ and $d\left(M;\tilde{p}(M,S,s),\alpha;P(M,S),x\right) = \alpha^\varepsilon\left(\frac{\tilde{p}(M,S,s)}{P(M,S)}\right)^{-\varepsilon} Y$. Here $Y = \frac{Mx}{P(M,S)}$ is the equilibrium determined real value of money holdings.

8.3 SREE: Linear Quadratic

For the case of linear quadratic preferences, the SREE defined in section 2.3 becomes a set of functions $\left\{\bar{p}(M), \tilde{p}\left(M;z,\alpha;x,\mu_Q\right), F^*\left(M;z,\alpha;x,disp_Q,disp_D,\mu_Q\right), P\left(M;x,disp_Q,disp_D,\mu_Q\right)\right\}$ such that:

- $\bar{p}(M)$ solves the *ex ante* pricing problem given the state dependent price index $P\left(M;x,disp_Q,disp_D,\mu_Q\right)$;

$$\begin{aligned}\hat{\varepsilon}\,\bar{p}(M)\,E_{\alpha;x,x',disp_Q',disp_D',\mu_Q'}\left[\frac{x'}{P\left(Mx;x',disp_Q{}',disp_D{}',\mu_Q{}'\right)}d\big(M;\bar{p}(M),\alpha;x\big)\right] = \\ E_{z,\alpha;x,\mu_Q}\left[\frac{d\big(M;\bar{p}(M),\alpha;x\big)}{\mu_Q z}\right]^2.\end{aligned} \tag{23}$$

- $\tilde{p}\left(M;z,\alpha;x,\mu_Q\right)$ solves the *ex post* pricing problem given the state dependent price index $P\left(M;x,disp_Q,disp_D,\mu_Q\right)$;

$$\begin{aligned}\hat{\varepsilon}\,\tilde{p}\left(M;z,\alpha;x,\mu_Q\right)\,E_{x',disp_Q',disp_D',\mu_Q'}\left[\frac{x'}{P(Mx;x',disp_Q',disp_D',\mu_Q')}\right] = \\ \frac{d\big(M;\tilde{p}(M;z,\alpha;x,\mu_Q),\alpha;x\big)}{\mu_Q{}^2 z^2}.\end{aligned} \tag{24}$$

- At the critical adjustment cost $F^*\left(M;z,\alpha;x,\mu_Q\right)$, the seller is just indifferent between adjusting and not:

  $F^*\left(M;z,\alpha;x,\mu_Q\right) = W^a\left(M;z,\alpha;x,\mu_Q\right) - W^n\left(M;z,\alpha;x,\mu_Q\right)$

- $P\left(M;x,disp_Q,disp_D,\mu_Q\right)$ is the aggregate price function in state $\left(M;x,disp_Q,disp_D,\mu_Q\right)$ given by:

$$P\left(M;x,disp_Q,disp_D,\mu_Q\right) = \left[E_{z,\alpha}(1-\Omega(F^*(M;z,\alpha;x,\mu_Q)))\bar{p}(M)^{(1-\epsilon)} + E_{z,\alpha}\Omega(F^*(M;z,\alpha;x,\mu_Q))\tilde{p}(M;z,\alpha;x,\mu_Q)^{(1-\epsilon)}\right]^{\frac{1}{1-\epsilon}} \tag{25}$$

Throughout, $d\big(M;\bar{p}(M),\alpha;P\left(M;x,disp_Q,disp_D,\mu_Q\right),x\big) = \alpha^{\varepsilon}\left(\frac{\bar{p}(M)}{P(M;x,disp_Q,disp_D,\mu_Q)}\right)^{-\varepsilon} Y$ and $d\big(M;\tilde{p}\left(M;z,\alpha;P\left(M;x,disp_Q,disp_D,\mu_Q\right)x,\mu_Q\right),\alpha;x\big) = \alpha^{\varepsilon}\left(\frac{\tilde{p}(M;z,\alpha;x,\mu_Q)}{P(M;x,disp_Q,disp_D,\mu_Q)}\right)^{-\varepsilon} Y$. Here, note that $Y = \frac{Mx}{P(M;x,disp_Q,disp_D,\mu_Q)}$ is the real output and thus real spending.

8.4 Quantitative Approach

We first present the idiosyncratic and aggregate shocks and continue with the menu cost parameterization used in the analyses. Then we conclude with the computational algorithm.

8.4.1 Shocks

Idiosyncratic shocks In the algorithm, idiosyncratic productivity shocks have unit mean and standard deviation denoted by σ_z.[34] Similarly, idiosyncratic demand shocks have mean of 1 and standard deviation, σ_d.

$disp_Q$ **Shock** When imposed, the spread of idiosyncratic productivity distribution itself becomes a stochastic process. $disp_Q$ shocks change the dispersion of idiosyncratic productivity shocks for everyone, therefore is an aggregate shock. When $disp_Q$ shocks applied, agents *ex ante* do not know whether they are going to draw their idiosyncratic productivity from a wider or a narrower distribution.

$disp_D$ **Shock** Similar to the $disp_Q$ shocks, when $disp_D$ shocks are alive young sellers do not know the dispersion in the idiosyncratic demand distribution.

μ_Q **Shock** μ_Q shocks moves the mean value of idiosyncratic productivity shock distribution, and similar to $disp_Q$ shock, when imposed *ex ante* agents do not know the mean value of the productivity distribution.

Combined μ_Q **and** $disp_Q$ **Shock** To impose perfectly negatively correlated combined shocks of $disp_Q$ and μ_Q, as in Vavra (2014) we associate the highest state of $disp_Q$ with the lowest state of μ_Q; the second highest state of $disp_Q$ with the second lowest state of μ_Q, than the third and *so on...*

8.4.2 Menu Costs

Firms are heterogeneous due to the realization of firm-specific price adjustment costs. Furthermore, firm heterogeneity stems not only from the realizations of menu costs, but we also impose another form of heterogeneity in the distribution of menu costs. A small fraction ψ of firms face zero price adjustment cost and thus have perfectly flexible prices. The remaining fraction $1-\psi$ draws from a nondegenerate distribution of adjustment cost. Figure 12 exhibits the shape of menu cost distribution.

The menu cost distribution follows Dotsey and Wolman (2019), using a tangent function given by:

[34] Given the one period nature of price setting, there is no gain to specifying the AR(1) for these shocks.

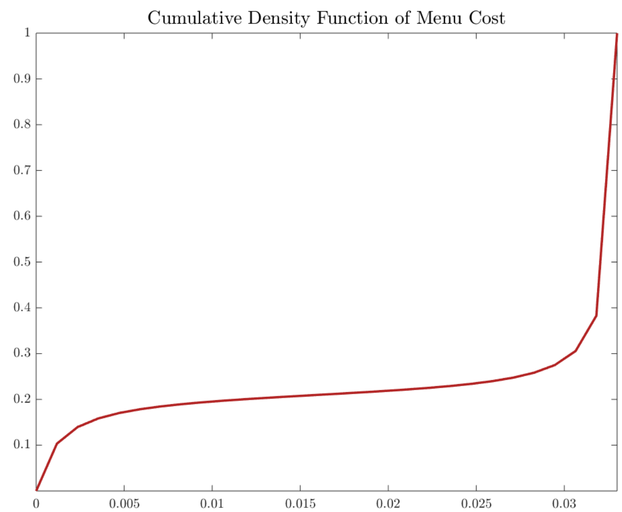
$$G(F) = \frac{1}{\omega}\left\{\tan(\frac{F-\kappa_2}{\kappa_1}) + \nu\cdot\pi\right\} \tag{26}$$

with

$$\kappa_1 = \frac{\bar{F}}{[\tan^{-1}(\omega-\nu\cdot\pi)+\tan^{-1}(\nu\cdot\pi)]}; \qquad \kappa_2 = \tan^{-1}(\nu\cdot\pi)\cdot\kappa_1. \tag{27}$$

The upper bound on the fixed cost, $\bar{F}$, controls the extent of price stickiness. As $\bar{F}$ increases, higher values for menu cost is now available, making the adjustment harder. The curvature parameters (ω,ν), are chosen so that $G(F)$ is monotonically increasing. As noted above, ψ governs the fraction of flexible-price firms, and thus increasing this value leads to a larger number of small price changes and a higher overall frequency of price adjustment. Corresponding values can be found in Table 1.

Figure 12: Menu Cost Distribution



Note: This figure shows the non-degenerate distribution of price adjustment costs.

8.4.3 Computational Algorithm

This section discusses how the SREE is computed. For the price setting component of the SREE, all of the state variables are exogenous except for the aggregate price level, $P(M,S)$.[35] In contrast, the aggregate price level is an equilibrium object, and is therefore calculated from the choices of the sellers, as in (25). Thus the focus of the solution approach is to find the equilibrium aggregate price function, $P(M,S)$ through the firm's individual price choices. For expositional purposes, we continue with the (S,s) notation as in Appendix 8.2, that is we define $S=(x,\mu_Q,disp_Q,disp_D)$ as the aggregate state space and $s=(z,\alpha,F)$ as the idiosyncratic state space.

Step 1 Start with an initial guess of the aggregate price function, $P^{(0)}(M,S)$.

[35]See Appendix 8.3 for the full definition of SREE in this linear quadratic setting.

Step 2 Calculate the new implied aggregate price function, $P^{(1)}(M, S)$, by solving the system. Specifically,

i. Solve the nonlinear system governed by (18) - (22). Note that (19) is not an independent equation *per se*, but a set of equations for each point in the state space. Solution to the system yields *ex ante* price, $\bar{p}(M)$ and *ex post* price set, $\tilde{p}(M, S, s)$.

ii. Using the *ex ante* price, $\bar{p}(M)$ and *ex post* price set $\tilde{p}(M, S, s)$, for each point in the state space, calculate the values of adjustment $W^a(M, S, s)$ and non-adjustment $W^n(M, S, s)$, given by (20) and (21) respectively.

iii. Compare the values of adjustment $W^a(M, S, s)$ and non-adjustment $W^n(M, S, s)$ for each point in the state space, and store the maximum value of each case and also record whether adjustment or non-adjustment yield this maximum value.

iv. Given the decision about price adjustment, pick the corresponding price (*ex ante* or *ex post*) and construct the realized price matrix for each point in the state space.

v. Given the probability of occurrences of each idiosyncratic state, calculate the new aggregate price matrix, $P^{(1)}(M, S)$ for each point in the **aggregate** state space.

Step 3 If the distance between $P^{(0)}(M, S)$ and $P^{(1)}(M, S)$ is within the error tolerance band, the aggregate price function converges yielding the price policy functions. If not, return to Step 1, update the guess of aggregate price function, *i.e.* $P^{(0)}(M, S) = P^{(1)}(M, S)$. Keep iterating until the aggregate price function converges.

Note that there is no approximation involved in the solution algorithm. The approach simply solves a system of equations to find a SREE. So unlike an approach based upon Krusell and Smith (1998), there are no moments *per se* used to characterize an equilibrium.